PHILOSOPHICAL

TRANSACTIONS,

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXIV.

PART II.

LONDON,

PRINTED BY W. BULMER AND CO. CLEVELAND-ROW, ST. JAMES'S;
AND SOLD BY G. AND W. NICOL, PALL-MALL, BOOKSELLERS TO HIS MAJESTY,
AND PRINTERS TO THE ROYAL SOCIETY.

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XV. *On a new principle of constructing His Majesty's Ships of War.* *By* Robert Seppings, *Esq. one of the Surveyors of His Majesty's Navy. Communicated by the Right Hon. Sir* Joseph Banks, *Bart. K. B. P. R. S.*

Read March 10, 1814.

NOTWITHSTANDING the rapid improvement in almost every other branch of the arts and sciences within the last century, it will scarcely be credited by persons not conversant with ship-building, that little or no advancement has been made during that period in naval architecture, so far as relates to the disposition of the materials which compose the fabric of a ship, whereby alone strength and fixedness of the parts can be obtained.

This will appear the more extraordinary in Great Britain, when it is considered that our very existence as a nation depends upon our naval superiority; and when it is further understood that a deficiency of oak timber, but more particularly that of a large scantling, calls for such an application of it as will reduce its consumption, and make up for the deficiency of its size.

It is not improbable, that the responsibility which would attach to an individual who should attempt an innovation in a

structure, whereby the lives of hundreds might be thought to be endangered, together with the great opposition raised against propositions for any material change in long established customs, may have occasioned this backwardness in naval improvements.

This heavy responsibility, together with the obstacles which are so frequently thrown in the way of projectors, has, without doubt, deterred many men, eminently distinguished for professional abilities, from attempting to carry into effect their ideas on this most arduous and dangerous undertaking: to which may be added, that not merely their professional reputation, but even their peace of mind, might have been hazarded thereby. For if the ship should be lost, although from a very different cause to that of the construction, yet would the weight of censure fall upon the new principle; and the projector would have to endure the whole burthen of the charge, that his schemes had occasioned not only the loss of the ship, but also, which is of far more consequence, the lives of some hundreds of his fellow-creatures. Whilst, on the other hand, should the experiment succeed, there would not be wanting those who would be ready to detract from that merit which he so richly deserved, and which would be purchased by him at so great a risk.

The writer of the present paper, in his various propositions for the improvement of naval architecture, has constantly kept in view these two leading axioms, "That the strength of a fabric consists not so much in the quantity of the materials of which it is composed, as in the disposition, the connection, and the security of its several parts." And, "that the strength of a ship, let its construction be what it may, can never

exceed that of its weakest parts," and consequently, "that partial strength produces general weakness."

Three 74 gun ships, now at sea, have already been rebuilt at Chatham on the principle about to be explained; and, from the favourable reports of those ships, the Lords of the Admiralty have given their orders for the building of several new ships upon the same principle.

To shew, in as clear a light as possible, the advantages of the application of this new principle to ship building, it may be necessary, for the information of those who are not acquainted with that art, to give the following general outline of the structure of a ship on the old principle.

1st. The frame of a 74 gun ship is formed of more than eight hundred different timbers, placed at right angles to the keel, which may be considered as the back bone of an animal, and the frame timbers its ribs. Each rib is composed of several pieces of the thickness of fourteen inches, or thereabouts. Between the several divisions of the frame, or ribs, is a space from one to five inches wide.

2dly. The whole exterior frame is covered with planks of different thicknesses, or to carry on the figure, the ribs are covered by a skin of greater or less substance from the extreme ends of them to the keel or back bone.

The inside of the frame is also almost entirely lined with planks; within which is another partial range, as it were, of interior ribs, at a considerable distance from each other, termed riders.

3dly. Across this frame are pieces of timber called beams, united together so as to be of sufficient length to reach from one side of the ship to the other.

The use of these beams is to secure the sides of the ship, so as to prevent her upper works from spreading, and to keep that part which is under water from being compressed by the fluid. They are also the supports or bearers of the decks (or what we call in houses the girders for the floors) and must therefore be of such strength as to endure the weight of the cannon, and whatever else is to be placed upon them. The usual mode of fastening these to the sides, has, generally speaking, been merely local, by two angular pieces of timber or iron (called knees) bolted to each beam, and also to the sides of the ship, by which means they were only partially held to the side, and there was wanting that continuity of materials, and consequently of strength, which the new system gives.

Between the beams and at right angles with them are placed pieces of wood called carlings, and at right angles with these (consequently parallel to the beams) ledges, which correspond with joists in a house. The planks or flat of the deck (flooring) is laid nearly in parallel lines from head to stern, upon and at right angles with the beams, and is fastened to them and to the carlings and ledges by bolts, nails, or wooden pins, called treenails. From this statement it will appear evident, that the decks, according to the old construction, are in nowise connected with the sides of the ship.

Having thus briefly described the common mode of shipbuilding, it will next be proper to point out such of its defects as the new principle tends to remove.

In the first place, it will be perceived, that all the materials composing the fabric of a ship are disposed nearly at right angles to each other.

This disposition which, in every wooden fabric is well known to the meanest mechanic, to be the weakest, is particularly so in a ship, the immense body of which, subject to violent action from impulses in every direction, is sustained by a greater pressure on the centre than the extremities, arising chiefly from the difference in the fore and after parts of the body, to that of the midship, or middle part.

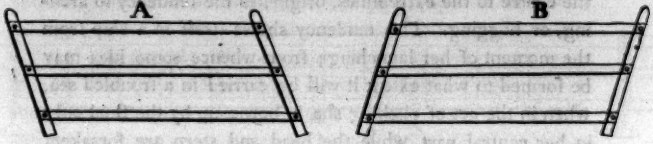
From the want of a continued succession of support from the centre to the extremities, originates the tendency to arching, or hogging. This tendency shews itself in a ship from the moment of her launching; from whence some idea may be formed to what extent it will be carried in a troubled sea, when in the act of pitching she is borne up by the fluid only in her central part, while the head and stern are forsaken, and therefore unsupported by the water.

If a straight line be drawn from the head to the stern of a ship, whilst on the slip, or in the dock, no sooner has she entered her own element than each end of this line will be found to have dropped from two to five or six inches, in consequence of the weakness of the fabric, and the two extremes wanting the quantum of support which the fluid gives to the central part.

The length of a 74 gun ship being 170 feet or more, it requires but little knowledge of the strength of timber to perceive that planking of that length, however thick, or in whatever way joined or put together, must, under the present system, bend with its own weight. The fastenings, and consequently the connection of the several parts of the fabric, must therefore suffer for the want of *stiffness*, and a change of form is the consequence.

This may be shewn by putting together four pieces of wood,

and securing them with iron pins in the form of a square; which on the least pressure may be made to change its form to the rhombus; but let another piece be fixed to it diagonally, and the figure of the frame will be found immoveable. Place a bar in the middle parallel to two of the sides, and secure it firmly by iron pins, still the figure will easily be moved by the hand, like a parallel ruler, and assume the rhomboidal shape of A or B; but apply to the frame what the



carpenters term the brace in a common field gate as Z, and the figure will remain, as before, immoveable.

If this brace or diagonal piece is not fixed to it, the outer part of the gate (or that part most distant from the hinges) will have a constant tendency downwards, until at length it will reach the ground.

Let fig. 1 and 2 represent two frames of wood composed of parts strongly connected by bolts or iron pins.

Fig. 1. Will represent the principle on which the present system of ship building is conducted.

Fig. 2. The new principle.

Let CC represent the fulcrum, or point of support.

And DD two weights attached for the purpose of ascertaining the comparative strength or stiffness of each frame.

This experiment will shew that the stiffness of fig. 2 is to that of fig. 1, as 6 is to 1, and the strength as 3 to 1.

The greater the length of the frames, the greater will be the advantages of the new principle, both in stiffness and strength.

Fig 1.

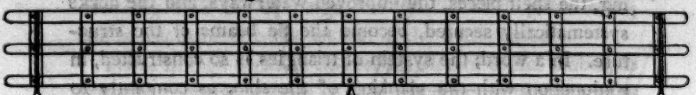
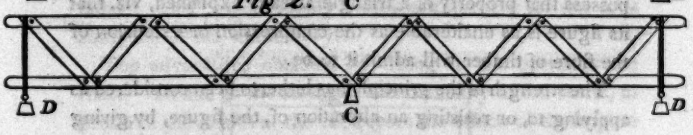


Fig 2.



The substitution of the triangle, as in the frame of fig. 2, for the rectangle in the frame fig. 1, comprehends the principle of the new system, the use and advantages of which will be sufficiently evident.

The arrangement of the materials in the triangular mode is such, that the pieces disposed horizontally are acted upon as ropes are by a strain of the fibre, whilst the other parts, composing a series of triangles, are pressed upon as pillars; in other words, the pressure acts in the direction of the fibres of the wood; whereas upon the rectangular, or old plan, the fibres are acted upon transversely, or across the grain, in the same manner as a stick is when placed across the knee and pressed

by the hands at each end, which first bends, and then breaks.

To prevent any transverse action upon the fibre of the timber, is one of the benefits arising from the new system, and to impede a longitudinal extension of the structure, is another.

For as the diagonal frame, composed of a series of triangles, aided by diagonal trussing between the ports, prevents the fabric from being acted upon transversely to the fibres of the materials horizontally placed, so the wales, the planking, the shelf pieces, the improved waterways, and the decks systematically secured, become the tie beams of the structure. In a word, the system of triangles is so constructed, in conjunction with the planking of the ship, as conjointly to possess that property of a triangle already explained, viz. that its figure is as unalterable as the compression or extension of the fibre of timber will admit it to be.

The strength of the principle has hitherto been considered as applying to, or resisting an alteration of, the figure, by giving great stiffness. It is now to be taken in another point of view, that of rendering the strength of the fabric as general and united as possible. For let it be again observed, the strength of any body is but equal to that of its weakest part.

In the new system, the openings between the ribs are filled in with slips of timber nearly to the height of the orlop, or lower tier of beams; which being then caulked, and paid or pitched over, makes the frame from head to stern, and within a few feet of the greatest draught of water, one compact and water tight mass of timber; so that were any of the outer planking of the bottom to be knocked off, the ship would not only still keep afloat, but would be secured from sinking. In

the old system the starting of a plank would be, and often has been, fatal.

The mode of filling in these openings between the frame, where the width of the space does not exceed three inches, is by driving in slices of wood cut wedge-like; two of which being driven, one from the outside, the other from within, form the parallel space of the opening, thereby bringing the parts into the closest contact. In the openings exceeding the width of three inches, the space is occupied by pieces corresponding with the openings, the fibre of such pieces being laid in the same direction as that of the frame timbers.

These fillings occasion no consumption of useful timber, as one fourth of the produce of slab, and other offal now sold as fathom wood, would supply a sufficient quantity for the consumption of the whole navy.

The advantages obtained by filling in the openings are these. To add to the strength and durability of the fabric, to preserve the health of the crew from the effects of the impure air arising from the filth which soon collects in these openings, to render the ship less liable to leakage, as well as to facilitate the stoppage of any leak, and lastly, to increase, as it may be said, the thickness of the bottom from four or four and a half, (the usual thickness of the plank) to about sixteen inches, thereby lessening very considerably the danger to be apprehended from getting on shore, or foundering at sea. That it tends also to the durability of the ship, will be inferred from the following positions.

1st. That the openings in the old principle are, after a ship has had any considerable length of service, choked up in many parts with an accumulation of filth.

2dly. That no free circulation of air can be obtained in these openings by any means.

3dly. That timber being either freely exposed to, or excluded from the air, is equally preserved.

4thly. That it has been found on examining the frame and plank of old ships, that those parts (now filled in), generally decay sooner than the rest, viz. from the floor-heads in the midships, and from the deadwood forward and abaft to the height of the orlop clamps.

If the above positions be true, it will follow, that by filling in these openings, much will be added to the durability of the ship; which also will be further promoted by omitting in these parts the inside plank, leaving thereby the surface of the frame timbers exposed to a free admission of air as often as the ship's hold is unstowed, and by the filling in, excluding the air from two of the sides of every timber.

By omitting the inside plank, much is added to the internal capacity of the ship's hold. For though the trussed frame projects from the timbers more by five inches than the thick-stuff at the floor heads, yet, as in the old system, the perpendicular riders are brought upon the thick stuff, their projection into the hold is more by eight inches than that of the new, the advantage therefore as to stowage is in favour of the diagonal frame. A tier of iron ballast will also be disposed of in this principle many inches lower, whereby an increase of stability will be given with less weight, which will favour the ship in carrying her ports higher out of the water, inasmuch as greater stability will be given with less ballast.

An accurate conception of the state of the ship's hold may be formed, by referring to the longitudinal section (Pl. X.

figure 1, which is termed the Jesuit's perspective, or bird's eye view) of the internal part of one side of a 74 gun ship in a complete state, with fillings in the openings between the timbers of the frames instead of the planking over them.

In this state the diagonal timbers are introduced, intersecting the timbers of the frame at about the angle of 45 degrees, and so disposed as that the direction in the fore, is contrary to that of the after part of the ship (as may be seen in the engraving), and their distance asunder from six to seven feet or more; their upper ends abutting against the horizontal hoop or shelf piece of the gun deck beams, and the lower ends against the limber strakes, except in the midships, where they come against two pieces of timber placed on each side of the keelson for the purpose of taking off the partial pressure of the main mast, which always causes a sagging down of the keel, and sometimes to an alarming degree. These pieces of timber are nearly as square as the keelson, and fixed at such a distance from it, as that the main step may rest upon them. They may be of oak, or pitch pine, and as long as can be conveniently procured.

Pieces of timber are next placed in a fore and aft direction over the joints of the frame timbers at the floor and first futtock heads, their ends in close contact with, and coaked or dowelled to the sides of the diagonal timbers. In this state the frame work in the hold presents various compartments, each representing the figure of a rhomboid.

A truss timber is then introduced into each rhomboid with an inclination opposite to that of the diagonal timbers, thereby dividing it into two parts. The truss pieces so introduced into the rhomboid, are to the diagonal frame what the key stone

is to the arch; for no weight or pressure on the fabric can alter its position in a longitudinal direction, till compression takes place at the abutments, and extension of the various ties.

This arch-like property of the diagonal frame, not only opposes an alteration of position in a longitudinal direction, but also resists external pressure on the bottom, either from grounding or any other cause, because no impression can be made in its figure in these directions, without forcing the several parts of which it is composed into a shorter space.

The connection which is kept up by means of this trussed frame firmly attached to the timbers of the ship by circular coaks and bolts, together with the shelf pieces united to the sides and to the several beams by means of the same sort of fastenings, gives such unity to the whole as to bear no comparison with that heterogeneous and badly connected mass of materials for which it is substituted.

It has hitherto been a generally received opinion, that stiffness or inflexibility in a ship is not strength, but that a yielding or bending of the fabric is an essential quality to preserve it from being destroyed by the shocks which it is destined to sustain.

This misconception must have arisen from an equally incorrect idea, which is, that a ship is an elastic body, because there is a considerable degree of elasticity in the materials of which it is composed. But it should be remembered, that this elasticity of the materials must be very inconsiderable, inasmuch as the minute degree of elasticity in each piece must necessarily be neutralised in the fabric, by the various directions and tendencies of the numerous parts of which it is

composed, so that a ship, let her construction be what it may, either loose or firm, is not in any case elastic. It follows then, that every action and reaction of the sea operating upon different parts of the fabric at different times, occasions, for the want of unity of the whole of the parts, a constant and increasing weakness, which by some may have been taken for elasticity.

When a sea strikes a ship forward, the bow will rise with the sea; which passing aft, lifts the midships in succession, leaving at this time, in a great measure, the fore and aft parts of the ship with comparatively little, or no support. Such shocks, acting upon a body whose parts are not firmly connected, produce a bending and rebending of the fabric, so that the several planks of the sides play over each other, and the fastenings are strained and loosened by a repetition of this action and reaction. On the contrary, when a body is constructed with such general unity and fixedness of all its parts, as that if one is moved the whole must move with it, then it may be said that all the parts of the structure bear their portion of the strain.

The decks come next under consideration, the beams of which are disposed in the new system nearly as usual, except that in midships, where a ship necessarily requires the greatest security, two additional beams have been introduced.

The beams of the several decks are attached to the ship's side in the following manner.

1st. By shelf pieces or internal hoops, distinguished by the letter E, figure 9. These shelf pieces are composed of several lengths of timber scarphed or joined together by coaks, or circular dowels, so as to form a kind of internal hoop, extending

from the hooks forward, to the transoms abaft, to the under-side of which, as well as the under parts of the beams, they are securely coaked, and being then firmly bolted to the side, instead of becoming a mere local fixture of the beam to the ship's exterior frame, as knees were, they are one continued and general security. The shelf piece is also a tie to the top-side in a fore and aft direction, cooperating with the trussed frame, as already explained.

2dly. By chocks, represented in figure 2, letter H., which are placed under all the shelf pieces in wake of the beams, except the orlop, in such a manner as to receive the up and down arm of the iron knees. The lower ends of those under the gun-deck shelf piece, step on the ends of the orlop beams, and those of the several decks above, step on the projecting part of the spirketting below. The chocks, particularly those between the orlop and gun decks, admit of their being driven into their respective places very tight, thereby acting like pillars. Another advantage attending them, is their great tendency to stiffen the ship's side, and to prevent the beam ends from playing on the fastenings when the ship is rolling, or straining under a press of sail.

The curved iron plate knees for securing the orlop beams, and the iron forked knees of the other decks, are described in figures 3 and 2.

The flat or planks of the several decks being, on the old system, each of them a mere platform, or in other words a cover to a box unconnected with the sides, are here so disposed of, as not only to oppose an alteration of figure from a force acting on the ship in a lateral direction, but also are made subservient towards securing the beams to the sides of the ship.

The framing and flat of the decks (excepting the quarter-deck, forecastle, and round-house, which are laid upon the old plan) are disposed of, as represented in figure 5. The former, that is, the framing or ledges and beams in ticked lines, the latter or planks in black; those of the starboard side being laid contrarywise to the larboard. The midship ends of the diagonal planks abut against two strakes laid in a fore and aft direction without side of the comings of the hatchways; the other ends approach the timbers of the frame, and the buts at each end are secured to a tier of carlings placed for that purpose. The flat or plank of the deck so disposed is connected with a certain number of coaks to the hooks, beams, and transoms. When the decks are thus laid, waterways, described in figure 2, are brought upon, and coaked to the ends of the plank. These waterways being then bolted through the ship's sides, and also, in an up and down direction, through the flat and shelf pieces, combine the whole in one homogeneous mass of strength.

Few ships are without some complaint of apparent weakness after three or four years service. These defects among other places shew themselves at the beam ends, which partial complaining proceeds in a great degree from the local attachment of the beams to the ship's side, and the flat or covering being in toto unconnected, as already explained. The extreme ends of the beams not being properly secured, play and work upon the fastenings, so that it is not unusual to see the bolt holes cut to an oval figure by the friction of the bolts. The remedy usually applied to a ship in this state is to load her with additional materials, such as iron knees, standards, breasthooks, &c. thus adding greatly to the original

weight of the fabric. Now it is evident, that the first gale of wind the ship encounters, after being thus partially strengthened, she must be reduced to the same state of weakness she was in before the remedy was applied.

This mode of strengthening ships may be compared to that of a raft firmly secured in the first place by strong lashing, which after some time works loose, or rather by working is stretched. As it might be too tedious a business to secure the raft by retightening the lashing, a small cord, or some twine, would be used to answer the purpose. It is clear that whilst the small cord or piece of twine remained tight, no part of the strain can bear upon the strong, but loose lashing, till the other stretches or breaks; so it is with a ship that has additional securities given her without refastening those which had worked, or were much strained.

To remedy defects, whether arising from the decay of the materials, or from any other cause, the principle now applied has many advantages, of which a slight inspection of the ship's hold (figure 1,) will convince, but in no respect is this advantage greater, than in the decks, for by shifting them when worn too thin for caulking, the original connection between the beams, decks, and sides, will be restored as perfect as at first.

The tendency of the ship to stretch or draw asunder in her upper works, being by no means obviated by the short planks on the inside between the ports, a truss piece of plank is substituted in lieu of them, which being well secured at the abutments, very materially aids the trussed frame, and gives great stiffness, thereby opposing the inclination to arch or hog aloft.

Figure 4 represents the stern of a ship with the trussing and iron work for its security. By this the helm port transom, which consumes one of the largest and most difficult trees required for a ship, is dispensed with.

Those essential qualities of strength, safety, and durability, having been detailed, a few observations with respect to the economy of the new principle may not be misplaced, which though but of a secondary consideration compared to the others, yet as the royal navy cannot be kept up without a supply of foreign timber, it evidently becomes a subject of considerable moment, that upwards of one hundred and eighty oak trees should be saved in a seventy-four, and a greater number in larger ships, allowing each tree to measure a load, or fifty feet rough contents.

The consumption of this scarce article may be further considerably lessened in the new system by the use of inferior, and old ship timber, which cannot be employed in the other, and if old ship timber was to be generally introduced, as was done in the Ramillies, one seventh part of the English oak required for a new 74 gun ship might be saved.

The facility of ascertaining the state, and making good the defects of the frame in the lower part of the ship, in consequence of omitting the inside planking, will also occasion a considerable saving of timber and workmanship, indeed the great ease by which any part of the diagonal frame may be replaced, justifies the making use of fir timber, particularly for the longitudinal piece and trusses.

But should the well grounded hopes of durability be realized, the saving of timber, and indeed of every article required for this enormous branch of the national expenditure, would be immense.

The author of this paper cannot conclude without observing that the appointment of most excellent and meritorious officers to the ships already completed on his principle, may be considered as a most favourable circumstance towards ascertaining the real merits of the construction.

Indeed, the orders for carrying this new principle of constructing His Majesty's ships into effect, were directed by such an honourable spirit of liberality; and so unshackled was the authority given, to enable the writer to carry his plans into execution, that no subterfuge can avail him should any failure be found in the system.

Fig. 4.

Plan of the Stern.

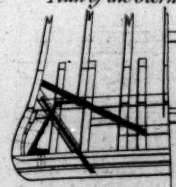
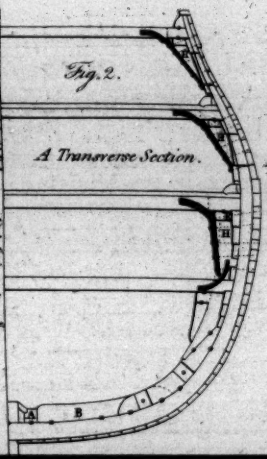


Fig. 2.

A Transverse Section.

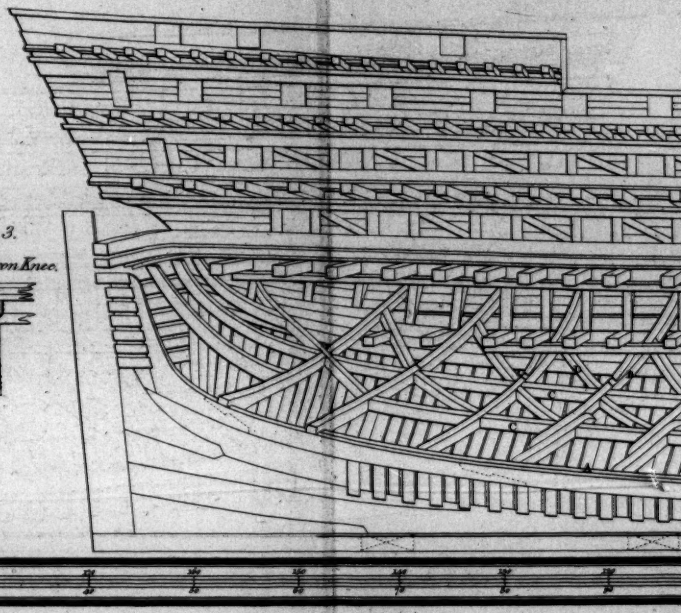


References.

AA. *Limber strake and additional Keelson forming abutments for the lower part of diagonal frame.*
BB. *Timbers of the diagonal frame.*
C. *Longitudinal pieces to Ditto.*
D. *Trusses to ..... Ditto.*
E. *Internal hoop or Gundeck shelf piece forming abutment for the upper part of the diagonal frame.*
F. *Abutment pieces for Trusses between ports.*
G. *Trusses.*
HH. *Chocks under Shelf piece for Iron Knees.*

Fig. 3.

Plan of Iron Knee.



Plan of the Breasthooks & Crutches.

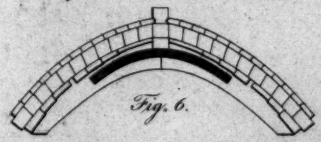


Fig. 6.

Fig. 6.

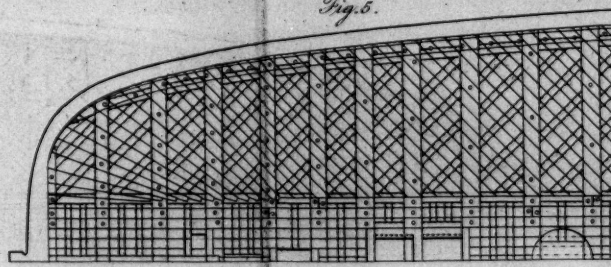
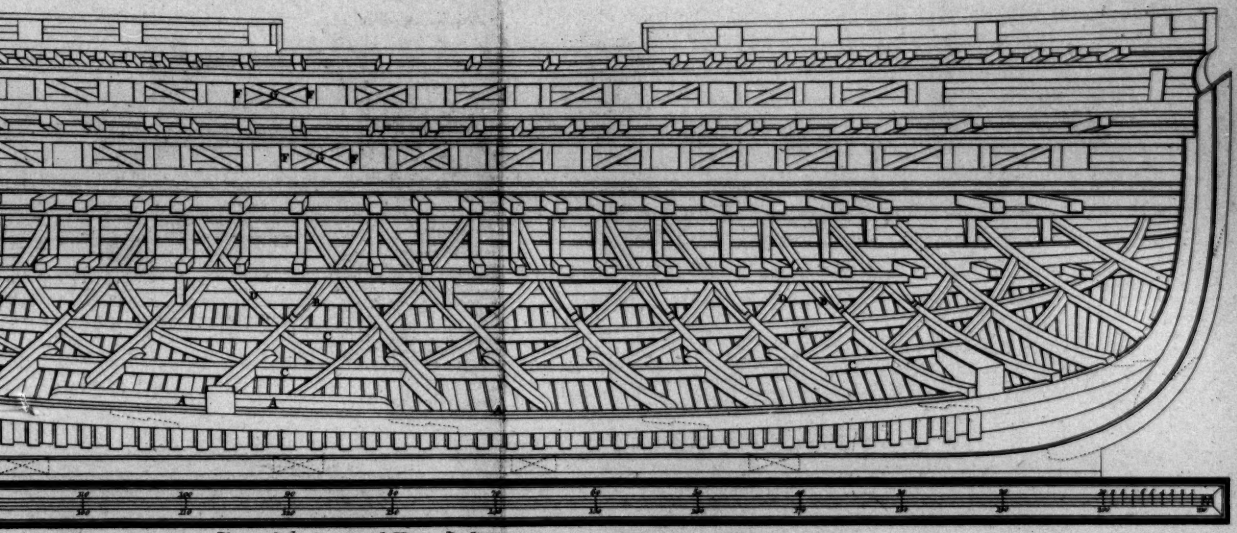
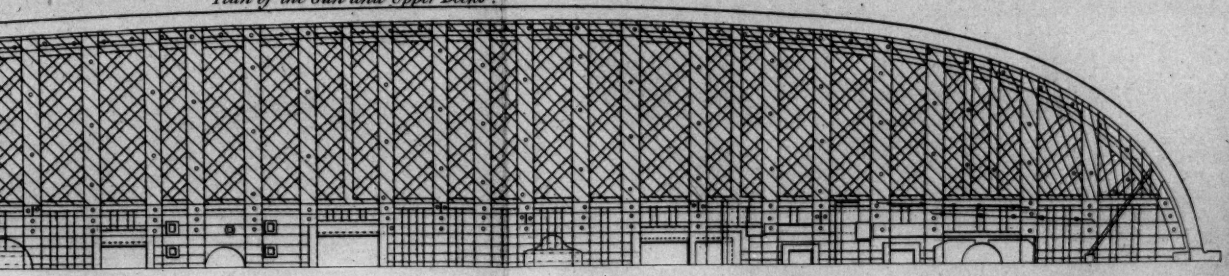


Fig. 1.

A Longitudinal Section.



Plan of the Gun and Upper Decks.



XVI. *Remarks on the employment of Oblique Riders, and on other alterations in the construction of Ships. Being the substance of a Report presented to the Board of Admiralty, with additional demonstrations and illustrations.* By Thomas Young, *M. D. For. Sec. R. S.*

Read March 24, 1814.

1. *Introductory Observations.*

The advantage derived from the employment of forces acting obliquely with respect to each other, in a variety of cases which occur in practical mechanics, has been demonstratively established by theoretical writers on the subject; and attempts have often been made to extend the application of the principle very considerably in the art of ship-building; but hitherto with very little permanent success. Mr. Seppings's arrangements are in many respects either new or newly modified; and the results of their actual employment, in the repair of the Tremendous, appear to be sufficiently encouraging to entitle them to a careful and impartial investigation, both with regard to the theory on which they are supposed to be founded, and to the facts which may be produced in their favour. The question, respecting the best disposition of the timbers of a ship, is by no means so easily discussed, as may be supposed by those, who have considered the subject but superficially; and if we allowed ourselves to be influenced by a few hasty arguments or experiments, we might be liable to the most dangerous errors: on the other hand, it may easily happen

that objections to the application of those arguments or experiments, which may occur at first sight, may be capable of being removed by a more minute investigation: and the importance of the subject requires, that no assistance, which can be afforded by the abstract sciences, should be withheld from the service of the public, even by those who have no professional motives for devoting themselves to it.

2. *Forces acting on a Ship.*

The first consideration that is necessary, for enabling us to judge of the propriety of any arrangement respecting the construction of a ship, is to determine the nature and magnitude of the forces which are to be resisted; and the second, to inquire in what manner the materials can be arranged, so as best to sustain the strains which these forces occasion. The principal forces, which act on a ship, are the weight of the whole fabric with its contents, the pressure of the water, the impulse of the wind, and the resistance of the ground or of a rock: and we must endeavour to ascertain the degree in which any of them have a tendency to bend the ship longitudinally or transversely, or to break through any part of her texture; and to inquire into those causes, which are likely to promote or to obviate the decay of the substances employed.

3. *Causes of arching. Weight.*

It is unnecessary to explain here the well known inequality of the distribution of the weight and pressure, which causes almost all ships to have a tendency to arch or hog, that is, to become convex upwards, in the direction of their length. It is possible that there may be cases in which a strain of a very

different nature is produced: but in ships of war, this tendency appears to be universal. It is however very different in degree in the different parts of a ship; and of course, still more different according to the different modes of distribution of the ballast and stores, which may occur in different ships: but in ordinary cases, it will probably be found nearly such as is represented in the calculations subjoined in the note,* de-

* In a modern 74 gun ship, fitted for sea, the length being 176 feet, the breadth 47½, the forces are thus distributed.

		Weight	Pressure	Difference
Aftermost	49 f.	699	627	72 tons.
Next	20	297	405	−108
	50	1216	1098	118
	20	290	409	−119
	37	498	461	37
	176	3000	3000	00

Now the laws of equilibrium will not allow us to suppose these forces concentrated in the middle of the respective portions, or equally distributed through them; and it becomes necessary, that one of the weights should be situated further forwards; which must be that of the foremost portion, containing the bowsprit and its rigging. It is also natural to suppose the excesses of weight and pressure distributed with as few abrupt changes as possible, so as to neutralise each other at the common termination of the adjoining portions, and to become more unequal in parts more remote from these neutral points. Thus the excess of weight in the first 49 feet being 72 tons, it may be supposed to begin at the rate of $\frac{144}{49}$ tons per foot, and to diminish gradually and equably, so that its centre of action will be at the distance $\frac{49}{3}$ from the end: the excess of pressure must increase in the next place, until at the distance of 59 feet from the stern, it becomes $\frac{108}{10}$ per foot, and must then diminish until it vanishes at 69, where the excess of weight must begin to prevail, becoming, at 94. $\frac{118}{85}$ per foot, and vanishing at 119. The excess of pressure might then be supposed to increase gradually through the next portion, in order to avoid an abrupt change at its extremity; but this supposition would still be insufficient, and it becomes necessary to imagine that for 6.6 feet the forces remain neutralised, and the pressure then prevails, so that its excess becomes at last $\frac{119}{67} = 17.7$ per foot: it must then decrease for 17.5

duced from data which have been obligingly furnished by an acute and experienced member of the Navy Board.

feet, and the excess of weight at the extremity must become 19.7 per foot, the neutral point being at 156.5. The equilibrium of the forces will then be expressed by the equation $72 \times 16.3 - 108 \times 59 + 118 \times 94 - 119 \times 134.5 - 155 \times 144.8 + 192 \times 169.5 = 0$, which is sufficiently accurate for every purpose.

From this distribution of the forces, we obtain a determination of the strain for each point of the respective portions, which is in the joint ratio of the magnitudes and distances of all the forces concerned, on either side of the point, reduced into a common result. For the first portion it is $\frac{144}{49} x \times \frac{1}{2} x - \frac{1}{2} \cdot \frac{144}{49} \cdot \frac{x}{49} \cdot x \frac{1}{3} x = \frac{72}{49} x^3 - \frac{1}{4} \cdot \frac{144}{49} \cdot \frac{x^4}{49}$, x being the distance from the stern: for the 2d, $72 (x - 16\frac{1}{4}) - \frac{1}{2} \cdot \frac{108}{10} \cdot \frac{(x-49)^2}{10}$: 3d, $72 (x - 16\frac{1}{4}) - 54 (x - 55\frac{1}{2}) - \frac{108}{20} (x - 59)^2 + \frac{1}{3} \cdot \frac{108}{100} \cdot (x - 59)^3$: 4th, $72 (x - 16\frac{1}{4}) - 108 (x - 59) + \frac{1}{4} \cdot \frac{118}{25} \cdot \frac{(x - 69)^3}{25}$: 5th, $72 (x - 16\frac{1}{4}) - 108 (x - 59) + 59 (x - 94) + \frac{118}{50} (x - 94)^2 - \frac{1}{3} \cdot \frac{118}{25} \frac{(x - 94)^3}{25}$: 6th, from 119 to 125.6, $72 (x - 16\frac{1}{4}) - 108 (x - 59) + 118 (x - 94)$: for the 7th, we must add to this expression $-\frac{1}{2} \cdot \frac{119}{13.4} \cdot \frac{(x - 125.6)^2}{13.4}$: and, in the last 37 feet, the strain will be expressed by $(176 - x)\, 19.7 \times \frac{1}{2} (176 - x) - \frac{1}{3} 19.7 \frac{(176 - x)^3}{19.5}$.

Hence we find the strain, at seven points, 22 feet distant from each other and from the ends, 605, 1993, 2815, 2244, 2655, 4610, and 1875; and by taking the fluxion of x in the seventh portion, we determine the maximum at $141\frac{1}{3}$ feet, amounting to 5281 tons, supposed to act at the distance of one foot.

In order to form an idea of the curve which would be produced by such a strain, acting on a uniformly flexible substance, we may consider the curvature as represented by the second fluxion of the ordinate y, and by finding and correcting the fluent separately for each portion, we may obtain the ordinate or fall at any given point corresponding to a given extent of arching of the whole fabric. It will however be sufficiently accurate for this purpose, to consider the forces as concentrated in a limited number of points, dividing those which act in the extreme portions into two parts, in order that the curvature may be continued to the ends; so that the whole of the forces may be thus distributed: at 0, 36; at $32\frac{1}{2}$, 36; at 59, — 108, at 94, 118; at 134.5,

Longitudinal Pressure.

To this strain another is added, from a cause, which, although not very inconsiderable, appears hitherto to have

— 119; at 144.8, — 155; at 163, 96; and at 176, 96. The strain for each portion may then be represented by $a - bx$, whence $\dot{y} = a\dot{x}\dot{x} - bx\dot{x}\dot{x}$, $y = ax\dot{x} - \frac{1}{2}bx^2\dot{x} + c\dot{x}$, and $y = \frac{1}{2}ax^2 - \frac{1}{6}bx^3 + cx + d$. It will be most convenient, in calculation, to make x begin anew with each portion, setting out from the middle, and to divide the numbers by 100, in order to shorten the operations: thus, for the middle portion, from 88 to 59, the strain will be $.2028 + .36x$, a being .2028, and $b = -.36$; and when x becomes .22, y is .00552, and when $x = .29$, $\frac{\dot{y}}{\dot{x}} = .0740$, and $y = .0011$; which values being substituted in the equations for the next portion, we have $c = .074$, and $d = .0011$: and by going through the whole length in this manner, we find the fall at the extremes and at seven equidistant intermediate points, .08697, .05325, .02514, .00552, 0, .00507, .02531, .06705, and .12325. If we wish to find the point at which the curve is parallel to the chord of the whole, we must inquire where $c = (.12325 - .08697) : 1.76$, which will be at 98 feet, or 10 feet before the midships.

We must next determine the magnitude of the strain arising from the longitudinal pressure acting on the lower part of the ship only. The resistance being supposed to be proportional in the first instance to the degree of compression or extension, according to the common and almost necessary law of the constitution of elastic bodies, and varying also in the direct ratio of the strength of the fabric, which may be assumed to be either equable, or, in the case of a ship, proportional to the distance from a point more or less remote, we must form an equation of equilibrium for the absolute equality of the forces in opposite directions, and another for their powers of acting with respect to any given point as the fulcrum of a lever. Thus the fluxion of the absolute resistance at the distance x from the upper surface, supposing the strength to be as $a + x$, and the neutral point, at which the compression and extension cease, to be at the distance b, will be $(b-x)\, e\, (a+x)\, \dot{x} = e\, (ab - ax + bx - xx)\, \dot{x}$, which, when x is equal to the depth d, must become equal to the force f producing the strain, or $f = e\, (abx - \frac{1}{2}ax^2 + \frac{1}{2}bx^2 - \frac{1}{3}x^3)$: and for the second equation, referring the forces to the upper surface as a fulcrum, the fluent of $e\, (b-x)\, (a+x)\, x\dot{x}$, must be equal to ef, e being the distance at which the force e is applied; whence $ef = e\, (\frac{1}{2}abd^2 - \frac{1}{3}ad^3 + \frac{1}{6}bd^3 - \frac{1}{4}d^4)$. Now if we make $a = d = x$, the equations become

escaped notice; that is, the partial pressure of the water in a longitudinal direction, affecting the lower parts of the ship only, and tending to compress and shorten the keel, while it has no immediate action on the upper decks. The pressure, thus applied, must obviously occasion a curvature, if the angles made with the decks by the timbers are supposed to remain

$c\left(\frac{1}{4}bd^3-\frac{1}{5}d^3\right)=f$, and $c\left(\frac{1}{4}bd^3-\frac{1}{15}d^3\right)=ef$, and from the former we have $c\left(\frac{1}{4}bd^3-\frac{1}{5}\frac{1}{4}d^3\right)=\frac{4}{5}df$; and, by subtraction, $\frac{1}{15}\frac{1}{4}cd^4=\left(\frac{4}{5}d-e\right)f$: consequently the force f may be considered as acting on a lever of the length $e-\frac{4}{5}d$: and if we take any other value for a, the fractional multiplier of d, instead of $\frac{4}{5}$, will be $\frac{3a+9d}{6a+5d}$, thus if $a=\frac{1}{2}$, we have $e-\frac{7}{1}\frac{1}{7}d$ for the length of the lever. In order to find the mean distance e at which the pressure of the water acts, we may suppose the form of the mean transverse section of the ship to be parabolic, and the area such as to correspond to the bulk of 3000 tons of water, each containing 35 cubic feet, the length being 176 feet, and the breadth 47½, whence the depth must be 18.84 feet: then the centre of gravity of a parabola being at the distance of ⅔ of the depth from the vertex, (VINCE'S *Fluxions*, p. 101,) and the centre of oscillation at ¾, when the point of suspension is at the vertex (p. 111,) the distance of these points $\frac{2}{3}\frac{1}{7}$ will be increased to $\frac{8}{3}\frac{1}{7}$, when the point of suspension is removed to the termination of the abscissa, and the distance of the centre of pressure from the vertex will be $\frac{1}{2}-\frac{8}{3}\frac{1}{7}=\frac{1}{7}$, and $\frac{2}{3}\times 18.84=8.074$, which, subtracted from $\frac{4}{7}\times 40=17.777$, leaves 9.703 for the length of the lever. Now the magnitude of the pressure on this section must be to 3000 tons, as the depth of the centre of gravity, 7.536 feet, to 176, that is, 128.45 tons, which, acting at the distance 9.703, will produce a strain of 1247 tons, or, in the terms of the preceding calculation, .1247, which is the multiple of $\frac{1}{8}x^3$ indicating the fall. These different causes of arching being independent of each other in their operation, their effects will be simply united into a common result: and the whole curvature of the ship, supposing its strength equable throughout its length, may be thus represented.

Dist. from the stern	0	22	44	66	88	110	132	154	176
Strain	1247.0	605	1993	2815	2224	2655	4610	1875	0
Fall	.04828	.02716	.01207	.00302	.00000	.00302	.01207	.02716	.04828
	.08697	.05325	.02514	.00552	.00000	.00507	.02531	.06705	.12325
	.13525	.08041	.03721	.00854	.00000	.00809	.03738	.09421	.17153
For 12 inches of arching	10.58	6.29	2.91	.67	.00	.63	2.93	7.37	13.42

unaltered, while the keel is shortened, in the same manner as any soft and thick substance, pressed at one edge between the fingers, will become concave at the part compressed, (Lect. Nat. Phil. I. Pl. 9. F. 117); and this strain, upon the most probable supposition respecting the comparative strength of the upper and lower parts of the ship, must amount to more than one third as much as the mean value of the former, being equivalent to the effect of a weight of about 1000 tons, acting on a lever of one foot in length, while the strain, arising from the unequal distribution of the weight and the displacement, amounts, where it is greatest, that is, about 37 feet from the head, to 5260, in a 74 gun ship of the usual dimensions; and although the strain is considerably less than this exactly in the middle, and throughout the aftermost half of the length, it is no where converted into a tendency to "sag," or to become concave. It must however be remembered, that when arching actually takes place from the operation of these forces, it depends upon the comparative strength of the different parts of the ship and their fastenings, whether the curvature shall vary more or less from the form, which results from the supposition of a uniform resistance throughout the length. An apparent deviation may also arise from the unequal distribution of the weight through the breadth of the ship: thus the keel may actually sag, under the step of the mainmast, even when the strain, as here calculated, indicates a contrary tendency with respect to the curvature of the whole ship.

Force of the Waves.

The magnitude of the strain on the different parts of a ship is subjected to very material alterations, when she is exposed

to the forces of the wind and waves. The effect of the wind is generally compensated by a change of the situation of the actual water line, or line of fluitation, so that its amount may be estimated from the temporary or permanent inclination of the ship; and the force of the waves may be more directly calculated from their height and breadth. These two forces can seldom be so applied, as to combine their effects, in producing a strain of the same kind in their full extent; it will therefore be sufficient for our purpose to determine the probable amount of the force of the waves, which is more materially concerned in affecting the longitudinal curvature than that of the wind. As a fair specimen of the greatest strain that is likely to arise from this cause in any common circumstances, we may consider the case of a series of waves 20 feet in height, and 70 in breadth; the form being such, that the curvature of the surface may be nearly proportional to the elevation or depression: a single wave might indeed act more powerfully than a continued series, but such a wave can scarcely ever occur singly. We shall then find upon calculation,* that the greatest

* The strain, produced by the pressure of waves of given magnitude, may be calculated from the comparison of the displacement with respect to their surface, compared with the displacement with respect to a level surface. It is true that the pressure upon the ship's bottom is not immediately derived from the temporary height of the nearest portion of water; but the horizontal motion of the water, which is the proximate cause of the elevation, is equally capable of affecting the fluid under a floating body, and of causing a pressure against it: the effect being nearly similar to the transmission of sound through an elastic medium. In other cases, the actual height of the fluid, over every particle concerned in the transmission of a wave, has been supposed, in calculations, to determine the pressure on it: but, whether from the necessary constitution of a fluid, or from the imperfect fluidity of fluids actually existing, it appears that there is a lateral communication of pressure within a certain angular extent, somewhat like the lateral friction attending the motions of fluids; and

strain takes place, in a 74 gun ship, at the distance of about 18 feet from the midships, amounting to about 10,000 tons, at the

this is the most probable cause of the deficiency in the velocity of waves, when their breadth is very small in proportion to the depth of the fluid. In the present calculation, however, the considerations are more simple, and we have only to determine the effect of the difference produced during the passage of a wave in the quantity of water displaced by the ship, with respect to the general surface. The total height of the waves being 20 feet, and the total breadth 70, the section being supposed to constitute a figure of sines, the elevation or depression, at the distance x from the middle, will be 10 cos. px, p being $\frac{6.2832}{70} = .08976$, the fluxion of the area $10\dot{x}$ cos. px, and the fluent $\frac{10}{p}$ sin. px; at the constant distance z from the middle, the fluxion of the p strain will be $(x - z)\ 10\dot{x}\varsigma px$, in order to find the fluent of which we must take the fluxions of $x\int px$, and ςpx, which are $\dot{x}\int px + xp\dot{x}\varsigma px$, and $-p\dot{x}\int px$; hence $\int x\dot{x}\varsigma px = \frac{1}{p}\, x\int px + \frac{1}{pp}\,\varsigma px$; and the fluent of the strain will be $\frac{10}{p}\,(x - z)\int px + \frac{10}{pp}\,\varsigma px + c$, which must vanish when $x = z$, so that $c = -\frac{10}{pp}\,\varsigma pz$: now, when $x = a$, the corrected fluent becomes $\frac{10}{p}\,(a - z)\int pa + \frac{10}{pp}\,\varsigma pa - \frac{10}{pp}\,\varsigma pz$; and if we take the fluxion of this, making z variable, we find, for the maximum, $-\frac{10}{p}\,\dot{z}\int pa + \frac{10}{p}\,\dot{z}\int pz = 0$, and $\int pz = \int pa$, so that z must be $a - 70 = 18$, whence the greatest strain is found, $\frac{10}{p} \times 70 \times .999 = 7791$, expressed in square feet of the longitudinal section, which, for a ship $47\frac{1}{2}$ feet wide, may be reduced into tons, by multiplying it by $\frac{47.5}{35}$, and will become 10572. It is true, that if the waves allowed time for the ascent or descent of the ship, so that she might float in equilibrium, the greatest strain would be little more than $\frac{2}{3}$ of this weight; but the elevating force in the case here calculated being only $\frac{1}{18}$ of the whole weight, it would require almost a second to raise the ship 1.265 feet, and to restore the equilibrium; so that notwithstanding its gradual application, dependent on the progressive velocity of the waves, which varies with the depth of the fluid, there must be an interval during which it operates very nearly in its whole extent, especially as the occurrence of a partial obstruction tends to increase the total height of a wave at the point where it is situated.

instant when the ship is in a horizontal position, while, in more common cases, when the waves are narrower, the strain will be proportionally smaller and nearer to the extremity. Hence it appears that the strain produced by the action of the waves may very considerably exceed in magnitude the more permanent forces derived from the ordinary distribution of the weight and pressure, being, according to this statement, nearly three times as great; so that when both strains cooperate, their sum may be equivalent to about 15,000 tons, acting on a lever of one foot, and their difference, in opposite circumstances, to about 5000. There may possibly be cases in which the pressure of the waves produces a still greater effect than this; it may also be observed, that the agitation accompanying it tends to make the fastenings give way much more readily, than they would do if an equal force were applied less abruptly. At the same time, it is not probable that this strain ever becomes so great, as to make the former perfectly inconsiderable in comparison with it, especially if we take into account the uninterrupted continuance of its action: it appears therefore to be highly proper that the provision made for counteracting the causes of arching should be greater than for obviating the strain in the contrary direction: for example, that if the pieces of timber, intended for opposing them were, on account of the nature of their fastenings, or for any other reason, more capable of resisting compression than extension, they should be so placed as to act as shores rather than as ties: although it by no means follows, from the form which the ship assumes after once breaking, that the injury has been occasioned in the first instance by the immediate causes of arching: since, when the fastenings have been loosened by a force of

any kind, the ship will naturally give way to the more permanent pressure, which continues to act on her in the state of weakness thus superinduced.

4. Breaking transversely.

The pressure of the water against the sides of a ship has also a tendency to produce a curvature in a transverse direction, which is greatly increased by the distribution of the weight, the parts near the sides being the heaviest, while the greatest vertical pressure of the water is in the neighbourhood of the keel. This pressure is often transmitted by the stanchions to the beams, so that they are forced upwards in the middle: when they are unsupported, the beams are more generally depressed in the middle, by the weight of the load which they sustain; while the inequality of the pressure of the water cooperates with other causes in promoting the separation of the sides of the ship from the beams of the upper decks. On the other hand, the weight of the mainmast often prevails partially over that of the sides; so that the keel is forced rather downwards than upwards in the immediate neighbourhood of the midships. The tendency to a transverse curvature is observable, when a ship rests on her side, in the opening of the joints of the planks aloft, and in their becoming tighter below; although this effect depends less immediately on the absolute extension and compression of the neighbouring parts, than on the alteration of the curvature of the timbers in consequence of the pressure.

5. Lateral Curvature.

In such a case there is also an obvious strain tending to produce a lateral curvature; and shores are sometimes employed to prevent its effects, when a ship is "hove down" on her side. This indeed is comparatively a rare occurrence; but when a series of large waves strikes a ship obliquely, they must often act in a similar manner with immense force: the elevation on one side may be precisely opposite to the depression on the other; and the strain from this cause can scarcely be less than the vertical strain already calculated: but its effects are less commonly observed, because we have not the same means of ascertaining the weakness which results from it, by the operation of a permanent cause. When a ship possesses a certain degree of flexibility, she may in some measure elude the violence of this force by giving way a little for the short interval occupied by the passage of the wave; but it may be suspected that her sailing, in a rough sea, must be impaired by such a temporary change of form.

6. Grounding.

When a ship takes the ground, she may either give way at once to the stroke of a rock, or rest on a bottom more or less soft, until she is either wholly or partially abandoned by the water. In the former case, her resistance must depend in great measure on the parts in the immediate neighbourhood of the injury: in the latter, it may happen, that she may be supported by so large a surface, as to be more in danger of parting aloft than of being crippled below. Commonly, however, the floor timbers are forced in at one end, the first fut-

tocks, which are their immediate continuations, being broken off; and sometimes the opposite ends of the floor timbers are forced out, especially in large ships without riders, their attachment to the keel remaining unimpaired.

7. *Decay.*

The causes which promote the decay of timber are only so far understood, as we are acquainted by experience with their effects. A partial exposure to moisture appears to be by far the most general of these causes: it is well known that total submersion does not accelerate decay; a surface which is kept moist by imperfect contact with another, so that a portion of water is retained between them by capillary attraction, seems always to be the part at which the timbers begin to rot; while both the surfaces completely exposed either to the drier air, or to the water, and those which are wedged closely together, and press strongly against each other, remain perfectly sound.

8. *Means of resistance.*

We are next to inquire into the comparative advantages of different angular positions of the timbers of a ship for resisting the forces which have been described; and in particular how far the arrangements, which have been proposed by Mr. Seppings, are better calculated for the purpose, than the common modes of construction. Whatever opinion we may ultimately form of these arrangements, they are by no means sufficiently justified by the experiments which have been exhibited in illustration of them. These experiments show, that when two parallel planks, (Plate XI., fig. 1,) have loose pieces interposed, extending perpendicularly from one to the other,

they are incomparably weaker, with respect to any transverse force, than when the intermediate pieces are in an oblique direction, so as to constitute a frame, which can only be bent as a whole. But it cannot for a moment be imagined, that the planks of a ship are connected with the timbers in as loose a manner as these transverse braces, which will scarcely support their own weight for the purpose of the experiment; and in fact the comparison would have required, that the whole space included by the parallelogram should be filled up in each case by similar braces, or at least that the two planks should have been firmly united at the loose end to the transverse braces (fig. 8); and it is demonstrable that in this case the same weight would have broken the pins, as if one of the planks had been oblique, or as if the planks had remained parallel, and had been connected by oblique pieces.

Such a result would, however, be far from proving the inutility of the addition of oblique braces to a rectangular frame: for the kind of strength, required for any particular purpose, is not always determined by the magnitude of the force which would be capable of breaking the substances concerned, although the power of resisting such a force is properly called strength, in the most limited sense of the term: but there are many occasions on which stiffness or inflexibility is of still greater consequence than strength, and others again on which flexibility is of material advantage. A coach spring, consisting of ten equal plates, would be rendered ten times as strong, if it were united into one mass, and at the same time a hundred times as stiff, bending only one hundredth of an inch with the same weight that would bend it a whole inch in its usual state, although nothing would be gained by the union

with respect to the power of resisting a very rapid motion, which I have, on another occasion, ventured to call resilience. (Lect. Nat. Phil. I. p. 143. II. p. 50.) Now it appears to be extremely difficult to unite a number of parallel planks so firmly together, by pieces crossing them at right angles, as completely to prevent their sliding in any degree over each other: and a diagonal brace of sufficient strength, even if it did not enable the planks to bear a greater strain without giving way, might still be of advantage, in many cases, by diminishing the degree in which the whole structure would bend before it broke.

The strength of a simple rectangular frame, firmly fixed at one end, is rendered somewhat less than double by perfectly fastening the joints at the other (fig. 4,) and the stiffness is nearly quadrupled.*

* When two horizontal bars are firmly fixed at one end only, and simply united at the other end by a vertical piece, their immediate joint force in resisting flexure remains unaltered; but if the vertical piece is firmly fixed to the ends of the bars, it may be considered as a lever held in equilibrium by four forces, arising from the repulsive and cohesive powers of the separate bars; and the sum of these forces must vanish when reduced to the same direction, while the sum of their actions, referred to any point as a fulcrum, vanishes also: and it is obvious that the total compression of the one bar will initially be equal to the total extension of the other, provided that their strength be equal. Hence, if the mean distance of the bars be a, and the depth b, reckoned between the centres of action of the respective forces, which in perfectly elastic bodies will be $\frac{1}{3}$ of the whole depth, the first force being x, and the second $-y$, the third will be y, and the fourth $-x$, and, from the equilibrium with respect to the point of application of the first force as a fulcrum, we have the equation $-by+xy-(a+b)\,x=0$, and $x=\frac{a+b}{a+1}y$, while the joint effect of all the forces in resisting the pressure of the weight is $2\,(y+x)\,b : c$, c being the length of the bars, or $\frac{2by}{c}\cdot\frac{2a}{a+1}$, while the resistance of the two single bars would be $\frac{2by}{c}$, the inclination of the elementary

The comparative security, obtained by the addition of a diagonal brace, is almost without limit. Supposing any number of planks of equal dimensions to lie simply on each other without any adhesion, and to be firmly fixed at one end, their aggregate strength will be very little greater than that of a single plank of one sixth part of the common depth or thickness of each, supported by a brace a little stronger, in the direction of the diagonal of the whole, (fig. 5); and the stiffness of the parallel planks will be as many times less than that of such a frame, as there are planks in one third of the whole series. Thus if we had twelve planks, six inches deep, and one thick, with friction rollers interposed, it is demonstrably true, however surprising, that they would be very little stronger in supporting a weight at the end, than a single tie an inch square in its section, assisted by a diagonal brace of equal relative strength: and also that this apparently slight structure would be nearly four times as stiff as the 12 planks, being depressed only one fourth as much, with a given weight, as the planks with a similar force acting on them.*

forces being here reduced to $\frac{b}{3c}$: and since the magnitude of y at the instant of breaking is given, the force will be augmented by the firmness of the connexion in the ratio of $2a$ to $a+b$, which is always less than that of 2 to 1. The stiffness may be nearly quadrupled by the fastenings, since the depression at the moment of breaking is reduced to little more than one half.

* "If one of the surfaces of a beam were incompressible, and the cohesive force of all its strata collected in the other, its strength would be six times as great as in the natural state." Lect. Nat. Ph. II. 50. Art. 335. Hence a plank of $\frac{1}{6}$ of the actual depth, acting simply as a tie, supported by a brace fixed at the distance of the depth, would be as strong as the original plank: and by increasing the distance of the point of support of the brace in the ratio of the number of planks, the strength of the two arrangements will remain equal, without altering the dimensions of the tie. The length of a plank being a, (Lect. II. 48. Art. 326.) the depth b, the height of the modulus

It is well known, that if the planks were firmly united into one mass, their strength would be rendered 12 times as great by the union, and their stiffness 144 times: but this is not the greatest resistance of which the materials are capable, even without any extension of their base of support; for if the planks were connected in pairs at half the distance of the whole depth, and allowed to move freely round fastenings perfectly secure, their strength, speaking theoretically, would be greater by nearly one half than if they formed a compact mass, while their stiffness would be only about one fourth as great: and an effect nearly similar might be produced if the respective pairs were united by oblique braces, extending over half the depth of the whole structure, although it would be very difficult, in practice, to make the strength of an arrangement of this kind even equal to that of a compact mass, since the fastenings could never be so perfect, as to bring every fibre of each plank into its full action at once, as the theory supposes. If the planks were already united into a compact mass, so as to be incapable of bending except as a whole, it is of importance to inquire whether any advantage would be

of elasticity m, the depression d, and the force applied at the end equal to the weight of a similar plank of the length g, we have $m = \frac{4e^3}{bbd}g$, and $d = \frac{4e^3}{bbm}g$; but, for a simple frame of two equal pieces, the force being g, the longitudinal extending force will be the weight of $\frac{e}{b}g$, and the actual extension $\frac{e^2}{b} \cdot \frac{g}{m}$, and the depression $\frac{2e^3}{bbm}g$, half as great as that of a plank of the same dimensions, when g is given, or supposing the weight on the frame sextuple, so as to be equal to that supported by the plank of six times the depth, three times as great; but by taking 12 planks together, we increase their stiffness only 12 times, while that of the frame is rendered 144 times as great by a similar extension of the base, so that it becomes in this case 4 times as great as that of the 12 planks.

gained by the further addition of oblique braces: and it will appear that if the braces were fixed to the outermost planks of the series only, they would have no manner of effect either on the strength or on the stiffness, whatever might be their direction; but if they were sufficiently fastened throughout their extent to each plank with which they come into contact, they would add both to the strength and to the stiffness, very nearly in the same degree as if they were fixed in the direction of the planks, at a distance from each other equal to their shortest actual distance, so as to constitute as many ribs as there are braces in a transverse line (fig. 6).* Hence, although there is obviously no economy in such an employment of oblique braces, yet it is by no means true that oblique braces are incapable of adding to the strength of a structure composed of pieces arranged at right angles; the assertion might however be very nearly correct in circumstances approaching to those of one of the experiments which have been exhibited for the purpose of illustrating the utility of such braces. On the other hand, the advantage of employing oblique braces must depend in great measure on the degree in which the angular position of the structure would be susceptible of variation without them; since, when properly fastened, they must universally tend to preserve the form unaltered,

* The extension and compression of the whole fabric being supposed equal, the diagonal braces will undergo no change of length, and therefore will not assist in the resistance, if only attached at their extremities. But in reality, although the extension and compression may be very nearly equal in the first instant of the change of form, the extension will always be much greater after a certain time, from the imperfection of the fastenings, which will allow the parts to separate, while their own resistance prevents their compression in a material degree: so that oblique braces, however fixed, must in this respect add considerably to the strength.

although they are somewhat less calculated to add to the ultimate strength of the principal tie or shore, than if their direction had been longitudinal. To take, for example, the case of a ship's arching or hogging: if the strength were overcome without any deficiency of stiffness, the upper decks and wales would be elongated, and the butts of the planks aloft parted, while the keel would be somewhat shortened, and the planks near it crippled, so that a ship 176 feet long and 40 feet deep, arching one foot with a uniform curvature, would have the length of the parts aloft, on the level of the quarter deck, 22 inches greater than that of the keel. If, on the contrary, the strength were not overcome, but the stiffness only failed, the angular situation of the parts being altered, and the joints simply becoming loose without parting, the planks would slide on each other, and their square ends would no longer remain in the same vertical line at the ports, while there would be no material alteration in the comparative length of the decks and keel, nor any permanent parting of the butts of the planks.

Grounds of decision respecting Oblique Riders.

This comparison therefore brings the question, respecting the general utility of oblique riders, into a very narrow compass; and we have only to inquire in what way it is most usual for ships to exhibit symptoms of weakness, in order to decide it. Now it will appear that, in cases of arching in general, some of the butts of the planks are always found to have parted aloft, at the same time that the angular position of some parts of the structure has as uniformly been more or less altered; and very generally a certain degree of sliding is observable in the planks at the sides of some of the ports.

This sliding is seen very distinctly in the planks of the Albion and of the Belliqueux, now at Chatham: at the same time there are also obvious indications of a certain degree of extension and compression: in the Albion, the butts of the planks have parted so far, that in some instances pieces have been let in between them; and in the Belliqueux, there is a space of about five inches between the middle of the deck transom and the carling, which had originally been in contact with it. In the Asia, lately launched in the Medway, the arching amounted to three inches and a quarter, and the comparative length of the upper and lower parts was probably altered about two inches at most: the parting of the butts amounting to $\frac{3}{16}$ of an inch each "for upwards of fifty feet in length in the midships, and for about eight feet from the top of the side," making a total extension of probably less than an inch: so that about half the effect seems to have been produced in one way, and half in the other; but apparently the greater half by the want of stiffness. It is also usually observable, that there has been some degree of permanent compression or crippling below, the butts of the planks opening when the cause of arching has been removed, and the sheathing being more wrinkled than would have happened from the simple bending of the planks. Where it has been observed, that the fore part of all the treenails supported the pressure of the planks in the after part of the ship, and the after part in the fore part of the ship, the observation must probably have been made on the lower parts of the ship, from the effect of a partial compression of this kind.

Authorities.

From this statement it appears, that unless some very strong facts can be produced, to disprove the probability, that the relative angular position of the parts constituting a ship may always be materially altered, without an absolute failure of strength, it cannot be denied that the principle of oblique bracing offers a remedy for the tendency to arch, whatever doubts there may be of the efficacy of any particular mode of applying it. And even if no observations could be produced in confirmation of the frequent occurrence of such a change of the angular situation of the timbers, the supposition that the stiffness could be perfect in this respect, notwithstanding the unequal shrinking of the timbers, and other similar circumstances, while the ultimate strength gave way by the failure of the fastenings, is in itself so highly improbable, that no positive evidence would be required for its complete rejection. We shall find accordingly, that Mr. Bouguer takes for granted the existence of a partial flexure, as sufficiently admissible without direct proof, and recommends the adoption of oblique planking as a remedy; and that other experienced authors have been equally favourable to the employment of some similar arrangements. In speaking of Mr. Gobert's mode of placing the cieling of a ship obliquely, Mr. Bouguer observes, that "this method cannot fail of producing the most desirable effects; for when the planking both within and without was arranged in the direction of the keel, it happened, in case of the ship's arching, that the rectangles formed by the timbers and the planking, merely changed their figure a little, so as to become rhombolds, two of the angles opening a little, while

the other two became more acute: but when the planks of the cieling are laid in an oblique direction, they serve as diagonals to the rectangles, so that a simple change of the relative angular situations of the sides is not sufficient to admit of the arching, without an alteration of the length of the diagonals, which would afford a resistance incomparably greater, especially at the upright parts of the sides, although at the floors it would have but little effect." Traité du navire, 154. Mr. GROIGNARD also, whose memoir, on the improvement of ship-building, has been obligingly communicated to me by an ingenious gentleman, formerly his pupil, although he objects to Mr. GOBERT's method, confesses that he "should have very much approved this mode of disposing the cieling, if it had been possible to employ straight planks, having the same obliquity without interruption, throughout the whole of the ship's length;" but thinks, with BOUGUER, that in carpentry, "every interruption is to be avoided as dangerous;" an objection so vague, as neither to require nor to admit a very distinct reply. Don GEORGE JUAN, too, after a calculation of the absolute strength of the pieces of timber employed in the construction of a ship, very properly remarks, that the effect of arching must be attributed not to their want of strength, but to "their play on each other."

9. *Mr.* SEPPINGS's *Braces.*

It appears therefore to be sufficiently established, that the principle of employing oblique timbers is a good one, provided that it be so applied as to produce no practical inconvenience. We must next inquire whether Mr. SEPPINGS has introduced it in a manner likely to be effectual, and not liable to any

material objections. He places, on the sides of a 74 gun ship, several series of oblique braces, principally between the ports, in the place of the internal planking, making an angle of about 24° with the decks; consisting of planks four inches thick, and about 11 wide, coaked and bolted to the timbers, and abutting against upright pieces similarly fastened. Now it follows, from what has already been stated, that these pieces have about four fifths as much effect in cooperating with the neighbouring parts, which act horizontally, as if they had been placed in the same situation with them, even on the supposition that the relative angular situation of the pieces is unalterably fixed: but for preventing the alteration of this situation, there is no doubt of their being very advantageously arranged, so far as their strength is sufficient; and the existence of a tendency to such an alteration, in a very material degree, appears to be altogether indisputable. Below the gun deck, the oblique timbers are considerably stronger, although they act under circumstances somewhat less favourable.

If, however, the resistance of a part of a structure is very immediately directed against a certain force, without an adequate cooperation from other parts of that structure, and if, being abandoned by those parts, it is exposed to a strain which it is too weak to withstand, it is obvious that it must inevitably be the first to give way, and must leave the rest of the fabric more exposed to be overpowered by such a force, than before its introduction. We must therefore inquire, how far it is possible that Mr. SEPPINGS's braces should be so abandoned. Now supposing a 74 gun ship to arch two feet, and one half of the change to depend on the sliding of the planks over each other, which will be allowed, by those who doubt the utility

of the arrangement, to be fully as much as can ever happen; the greatest fall of the surface will be one foot in 44, and the length of the brace will be diminished $\frac{1}{1188}$, or $\frac{7}{10}$ of an inch in the length of six feet, which, with a moderate allowance for the partial yielding of the fastenings, it will be perfectly capable of supporting without being crippled, although indeed it could scarcely support much more. It is obvious, however, that this supposition in many respects far exceeds the utmost that can possibly happen: and it would even require a greater force to produce such an effect on the braces, than any which the ship actually sustains. In order to calculate the magnitude of the greatest strain which these pieces could support, it will be safest to proceed on the supposition, that each square inch of the section of good oak timber is capable of resisting the pressure of four tons on an average: it will then appear that a single series of such braces, as Mr. SEPPINGS employs, extending throughout the length of each side of the ship, would support a weight of 143 tons, in whatever way the force counteracting it might be applied; and estimating the effect of all the braces and riders as equivalent to about four such series, the whole would resist a force of 570 tons; while the greatest force derived from the distribution of the weight, together with the action of such waves as we have considered, amounts to about 450 tons: so that the strength of these braces can scarcely be insufficient to support the pressure, unless the ship should be left dry, resting on the middle of her keel, and the braces should be abandoned by all the other parts, which usually cooperate with them.* The fastenings must indeed

* If a jointed parallelogram, composed of pieces of invariable length, having one of its sides fixed in a vertical position, be supported by a diagonal brace, the compres-

be considerably weaker than this, and the other parts of the ship considerably stronger; but since the fastenings appear to possess sufficient strength to resist any strain which is actually likely to affect them, there seems to be no inconvenience in their inferiority to the other parts. In fact, the Tremendous actually supported, for three days, without any perceptible change of form, a strain fully equal to that which is here calculated, having been purposely left on shores, which extended through 52 feet only of her length. But it must be remembered, that such a force, from its very gradual application, must be much less trying to the ship's strength, than the more abrupt changes which occur at sea, and it must on the whole be inferred, that it would be unsafe to trust to the braces alone, unsupported by the cooperation of the neighbouring parts. It would probably be easy to add some further strength to these braces near the ends of the ship, where the strain on them is the greatest, especially about 30 feet from the head, if it were found that they gave way before the rest of the timbers; and it might also be possible to replace them, if they

sion or extension of the brace will be to the descent of a weight connected with the moveable end of the parallelogram, as the depth of the parallelogram to the length of the brace, whatever the actual distance of the weight may be; so that although the strain on the horizontal pieces increases with this distance, that which affects the brace is independent of it; the relative being to the absolute strength as the depth of the frame is to the length of the brace. We must therefore inquire, what is the greatest absolute force that can be supposed to urge a given portion of the fabric in either direction: thus the excess of weight which has been attributed to the bowsprit and the neighbouring parts being 192 tons at 19½ feet from the head, this force may be occasionally increased by a similar pressure derived from the effect of the waves, which alone would amount to 302 tons at 35½ feet from the head, and which may sometimes cooperate with the former, so as to constitute a force of about 450 tons, about 25 feet from the head.

had once failed, with greater ease than many other parts of the fabric.

It may be questioned how far it is allowable to omit any part of the inner planks between the ports, for which the braces are a substitute, on account of their utility in securing the butts of the planks, which are always made to shift where they are supported by this subsidiary tie: but with the outer planking which remains, and with the partial assistance of the braces, to say nothing of that of the shelf pieces, it can hardly be believed, that the tie is more likely to part between two ports of the same deck, than immediately over one of them.

It has been very ingeniously observed, that arching is not only a part of the evil occasioned by a ship's weakness, but that it has an immediate tendency to afford a partial remedy for the cause which produces it, by making the displacement greater at the extremities of the vessel, and smaller in the middle: but, in fact, this change appears to be too inconsiderable in its extent, to produce any material benefit: the strain at the midships being diminished by each inch of arching only 66 tons, supposed to act at one foot: so that very little relief is obtained from the change, in comparison with the whole strain.

The case of the Kent, which broke in a remarkable degree, notwithstanding the employment of riders of large dimensions, is perfectly reconcileable with the principles which have been laid down: indeed these riders, which made an angle of a few degrees only with a vertical line, could have so little effect either on the strength or on the stiffness of the structure, that there was not the slightest reason to expect any material advantage from their application.

The explanation which has been given of the universal tendency of ships of war, in all common circumstances, to arch throughout their length, is sufficient to justify the different directions in which Mr. SEPPINGS now arranges his braces in the different parts of the ship, since they must necessarily afford a greater strength as shores than as ties, and since the most permanent and the greatest strain will generally be such as to call them into action in this capacity. When, however, a ship is compared to an inverted bridge, it must not be forgotten how necessary it frequently becomes, to consider these braces in a different capacity, and to provide for this contingency, as indeed Mr. SEPPINGS has not neglected to do, by employing such fastenings, as are extremely well adapted to secure their action as ties.

The shelf pieces, which Mr. SEPPINGS employs, and the superior strength of the fastenings of his decks to the breast hooks and transoms, have so obvious a tendency to counteract the causes of arching, that it is unnecessary to insist on their utility: the weight and expense of the shelf pieces are probably the only drawbacks upon the advantages, which they are so manifestly calculated to afford, in resisting both a vertical and a lateral strain; and even in these respects, they appear to be preferable to the wooden knees formerly employed.

The filling up the intervals of the timbers, throughout the hold, with wedges of old stuff, is perhaps the most indisputably beneficial of all the alterations which Mr. SEPPINGS has either introduced, or revived in an improved form. The strength, which is thus obtained, acts immediately in the prevention of arching, and is probably, in this respect, more than an equivalent to that of the internal planking, which has been

omitted; while the cohesive strength of the external planking, considered as a tie, is still probably more than sufficient for resisting the smaller force, which occasionally operates in a contrary direction: although the strength of the ship, for resisting such a force, is certainly much diminished by the change. From the manner in which these wedges are driven by Mr. SEPPINGS, it may easily be understood, that they may tend to produce a convexity below, without raising any part of the keel from the blocks, merely causing it to press more strongly on them at the midships; so that if this difference becomes equal to that of the weight and pressure after launching or floating, there may be no tendency to any further change whatever; and hence it may happen, that without any other superiority of stiffness, or even of workmanship, a ship may appear wholly exempt from arching, as the Tremendous did, and some other ships are said to have done. Without the operation of some such cause, even a hollow cylinder of compact oak, 180 feet long, 50 feet in diameter, and six inches in thickness, if such a mass could be supposed to exist, would exhibit, when immersed to the depth of its axis, a degree of arching just perceptible, from the longitudinal pressure of the water only, amounting to about $\frac{1}{16}$ of an inch;* besides a

* "The stiffness of a cylinder is to that of the circumscribing prism, as three times the bulk of the cylinder is to four times that of the prism." (Lect. Nat. Phil. II. 83. Art. 339. B.): but the radius of curvature of a prismatic beam is $\frac{bbm}{12af}$ (P. 46. Art. 321.) b being the depth, m the weight of the modulus, f the force, and a the distance of its application: and taking m for the weight of the modulus of the cylinder, its radius of curvature will be $\frac{bbm}{16af}$. But since the stiffness is as the 4th power of the diameter, (P. 49. Art. 333.) that of the hollow cylinder in question will

curvature proportionally greater from the other strains, which have been already calculated. Mr. SEPPINGS has also very properly introduced, in the Tremendous, an additional kelson on each side of the step of the mainmast, in order to support its weight, and to prevent the partial sagging of the keel.

10. *Riders.*

With respect to the transverse strain, or the tendency of the sides to sink in comparison with the keel, some strength is probably gained by Mr. SEPPINGS's mode of fixing the filling timbers in the same manner as the frames: and some advantage must be attributed to the cooperation of the wedges, or fillings in, with the timbers, as far as their connexion is capable of bringing them into action. The common cieling is by no means advantageously placed for assisting in a resistance of this kind, since it can only act where the curvature would be increased by the bending of the sides, and even there can only be compressed in a transverse direction (fig. 7). The riders

be reduced in the ratio of 1 to $1 - .98^4 = .0786$. Now when a cylinder is immersed to the depth of its axis, the calculation of the effect of the longitudinal pressure exactly resembles that of the stiffness, the strain being to that which would be the effect of the pressure on the ends of the circumscribing prism as $\frac{2}{\pi} \times .7854 = .58905$ to 1: but the strain on the prism would be $= 50 \times 25 \times 12.5 \times \frac{50}{3} : 35 = 7440.5$, and for the cylinder, $af = 4383$: and since the height of the modulus of elasticity of oak is 5060000 feet, (p. 509), and its specific gravity nearly equal to that of water, or perhaps a little greater, we have $m = 5060000 \times 50 \times 50 \times .7854 : 35$ tons, and the radius of curvature $.0786 \frac{bbm}{16af} = .0786 . \frac{50^4 \times 5060000 \times .7854 : 35}{16 \times \frac{2}{\pi} \times .7854 \times 50^4 : (24 \times 35)}$ $= .0786 . \frac{5060000}{12 : 24} = 795432$ feet; and dividing the square of 90 by twice this number, we have .0051, or one sixteenth of an inch, for the versed sine or arching.

commonly placed upon it, on the contrary, are very favourably situated for assisting in this action; but Mr. SEPPINGS's riders are so much more numerous, as to possess, notwithstanding their obliquity, a still greater force. The fastenings of the beams to the sides are also concerned in resisting a strain of this kind, as well as in counteracting the tendency to an extension aloft, which is the more immediate consequence of the unequal pressure of the water against the ship's sides. Mr. SEPPINGS's fastenings, so far as they depend on the shelf pieces, have probably some advantage over the more common ones; but the iron knees, which he employs (fig. 8.) do not appear to be quite so economically arranged as the straps of a simpler form, which other builders have used; they afford indeed a very direct connexion with the timbers, and they save some valuable wood in the chocks which support them: but still there appears to be some waste of strength when they act as ties, from the great obliquity of the shoulders, with respect to the direction of the force; to say nothing of the expense of the workmanship: and if, as Captain CAMPBELL seems to have suspected, there is any slight deficiency in the transverse strength of the Tremendous at the waterways, the circumstance may afford a further reason for doubting of the utility of these fastenings.

11. *Decks.*

The least obvious advantage attributable to the obliquity introduced by Mr. SEPPINGS appears to be in his mode of laying the planks of the decks; parts which seem to be principally required to cooperate with the sides of the ship as ties in a longitudinal direction: for the slight curvature, which is

given to them, can no more render them incapable of such an action, than the bending of a towing rope prevents its pulling along a boat. But in the first place, the lower decks can have little or no action of this kind, from their near approach to the line, at which extension ceases, and compression begins, at least until some of the fastenings give way; and secondly, the upper decks lose but one third of their strength in this capacity, by having their planks disposed at an angle of 45 degrees with the sides, while the obliquity must be capable of affording some additional power of resisting the violent action of the waves, which sometimes produces an immense strain in a transverse or lateral direction, as well as of enabling the ship, in case of necessity, to be more safely "hove down" on her side. There seems also to be some convenience in having the ends of the planks covered by the waterways, with respect to keeping the wings of the ship dry, although it has been suspected that the ends so covered may be rendered somewhat more liable to decay. It may, however, be apprehended, that any force, tending to shorten the deck, will have some little effect in forcing out the sides; for instance, if the whole deck became three inches shorter, the length of the planks remaining the same, they must force out each of the sides about a quarter of an inch, provided that their connexion with the beams allowed such a change, which appears indeed somewhat improbable. There may possibly be a slight difficulty in adjusting the planks to the curvature of the beams; but this difficulty appears to be readily overcome in other cases, as in that of the common cieling. It may hereafter deserve to be inquired, how far an oblique direction of the carlings between the beams, which in their present situation seem to contribute very little

to the strength, might enable them to cooperate in resisting a lateral force, if the arrangement could be made without too much weakening the beams, in procuring proper abutments for these pieces.

12. *Floors.*

It cannot easily be admitted, that Mr. SEPPINGS's construction affords any additional strength to a ship's bottom in case of her grounding. The fillings in between the timbers must indeed be extremely useful in this respect, first by giving firmness in the direction of the length, since even a straight plank is strengthened by having the incompressibility of its outside increased, much more one that is curved, in however slight a degree; and secondly, by cooperating with the timbers, considered as shores, so far as the wedges are fixed in their places by their lateral adhesion or otherwise.

The cieling, which has been omitted, can have very little effect by its own strength in preventing the separation of the timbers at the floor heads; but where there are transverse riders, it must be of essential advantage in enabling these to come into action, for the support of the neighbouring parts exposed to pressure; somewhat more effectually indeed, in many cases, than Mr. SEPPINGS's diagonal riders and their trusses can do, notwithstanding the superiority of their number and aggregate strength; on account of the magnitude of the intervening spaces, which might happen to receive the principal stroke near their centres. This magnitude does not, however, contribute by any means in the same proportion to the weakness of the parts, as it would do if the surface were plane: and it is not improbable, that for supporting the weight of the ship on a very soft ground, Mr. SEPPINGS's arrangement might afford equal strength with the common form, as seems

to have been exemplified by his experiment of leaving the Tremendous for three days on fourteen shores only, without injury: but for encountering the stroke of a rock, or of very hard ground, Mr. SEPPINGS's ship would probably be inferior, since in this case greater stiffness, even with equal strength, would be detrimental rather than beneficial; while, on the other hand, she would undeniably be less liable to suffer from any injury that might happen to her outer planking only; and, from her superiority in this respect, might possibly sustain, without inconvenience, a stroke, which would be ultimately fatal to a ship of a different construction.

18. *Durability.*

There does not seem to be the slightest ground for the apprehension, that the filling in should render the ship's timbers liable to decay: on the contrary, the timbers of the Sandwich were found perfectly sound in the lower half of their length, opposite to the wedges which had been driven in between them, and completely decayed in the upper half, where they had been exposed, in the usual manner, to the action of the confined moist air and water; and this result is perfectly conformable to analogy with the few facts that have been ascertained, respecting the general causes of decay. The utility of the filling in, for preventing the accumulation of filth, and for keeping the ship free from foul air, with respect to the comfort, and perhaps to the health of the crew, is too obvious to require discussion. How far the economy of timber may in all cases be so great as Mr. SEPPINGS is disposed to believe, can best be ascertained by those who are in the habit of estimating its value: but if the durability of the vessel

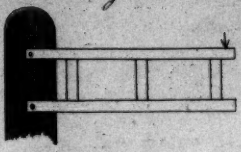
only were improved, at an equal expense, the adoption of his alterations would still be highly advisable.

14. *Conclusion.*

It is by no means impossible, that experience may suggest some better substantiated objections to these innovations, than have hitherto occurred: but none of those objections, which have yet been advanced, appear to be sufficiently valid to warrant a discontinuance of the cautious and experimental introduction of Mr. SEPPINGS's arrangements, which has been commenced by orders of the Board of Admiralty. The filling in seems to be wholly unexceptionable: the braces between the ports appear to be decidedly more beneficial than the planks for which they are substituted; and the coakings seem to be very judiciously employed in various parts of the structure: but something more may possibly be hereafter effected for the further improvement of the decks, and for the more complete provision of a substitute for the thick stuff of the cieling, in addition to the diagonal riders, if experience should prove that there is any deficiency in the resistance of these parts. But it must be remembered, in forming conclusions from such experience, that when an arrangement of any kind has nearly attained the maximum of its perfection, it may demonstrably be varied in a considerable degree, without a proportional alteration of its effect; so that the most correct knowledge of scientific principles, and the minutest accuracy in their application, must become indispensably necessary, in order to secure us from the introduction of material errors, derived from the latent operation of accidental causes, foreign to the immediate subjects of investigation.

Welbeck Street, 30 Dec. 1811.

Fig. 1.



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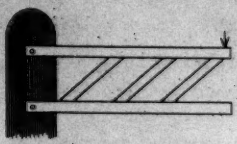

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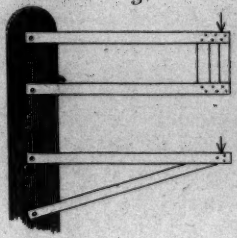

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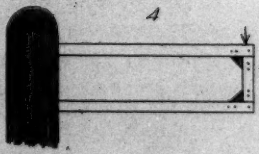

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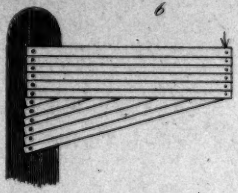

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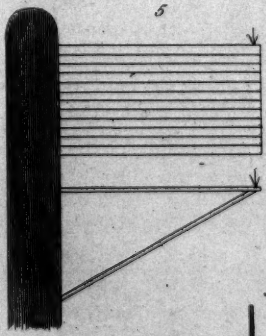

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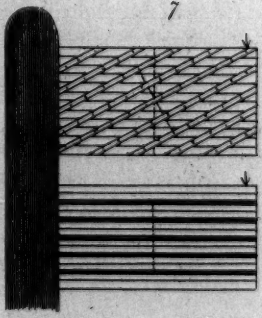

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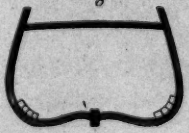

XVII. *Some further Observations on Atmospherical Refraction.* *By* Stephen Groombridge, *Esq. F. R. S.*

Read March 31, 1814.

In my former paper on atmospherical refraction, communicated to the Royal Society by my late friend, Dr. Maskelyne, I considered the few observations made below 80° of zenith distance, as not sufficiently to be depended on, for the computation of a general formula of refraction: and I therefore used η Ursæ Majoris (78° 10′ zen. dis.) as the lowest star for that purpose. Having since applied the computed refraction from the formula thence obtained, to observations of stars below 80°, I have noticed, that such stars so corrected, appeared to be further from the zenith below the Pole, than they ought to have been, from the observations above the Pole: and therefore that the refraction was less at those distances from the zenith, than I had assumed. This has induced me, in the years 1811 and 1812, to make a course of observations of stars below the Pole, above 80° zenith distance; and as near to the horizon, as the trees in Greenwich Park would permit; these being higher than the level of my Observatory. It may also be remarked, that those stars in my former table below 80°, produce the co-latitude in excess; as a confirmation, that the same formula will not apply to those larger arcs, where, from the rapid increase of the tangents, a small error in the assumed quantity becomes more sensible. Although various hypotheses may be formed, from the known density and tem-

perature of the atmosphere; and from these causes may be computed the effect they should have on a ray of light passing through the same: yet we must resort to observation, for the verification of the theory; and reduce the quantity so found, to the most simple and convenient formula. I shall proceed to deduce, from this course of observations, such formulæ as will appear to result, for the computation of the refraction; from the zenith, to the lowest star which I have observed: these may be considered as sufficient for the observation of the sun at the winter solstice, in high latitudes; since those of the moon, from its great parallax, and the planets from their general invisibility, would probably not be attempted. Nevertheless, it is to be wished, as a matter of curiosity, or from which some useful deductions might be made, that in those Observatories, wherein from their elevated situations it might be practicable, the true quantity of refraction should be ascertained to the horizon.

Of all the formulæ for computing the mean refraction, that proposed and used by Dr. BRADLEY, is the most convenient and applicable for the practical astronomer. But as it is now acknowledged, that the numbers he had assumed for the coefficient of r (the refraction;) and of x (the quantity at 45°) were too small: their real values will appear to be the mean of several arcs, and such as I now propose to be adopted. I have found, that the same formula will serve to 87° of zenith distance; possibly this might not happen in low situations, where the height of the vapours would form a greater angle with the horizon: yet in more elevated places, we may reasonably suppose, that a general formula might be carried nearly to the horizon.

In the two annexed tables are given the mean of the observed zenith distances of the several stars; and in the next column the mean refraction, computed from the formula which I proposed in my former paper; viz. tang. $z - 3.3625\, r \times 58'', 1192$: and which has been applied to these observations, corrected for the barometer and thermometer. In the following column, the error or difference is shewn, between the computed and real zenith distances; the assumed mean refraction, corrected by these errors, will give the mean refraction, which should have been applied. From these last quantities are deduced the respective values of (y) the coefficient of r, and of x: and from the mean of the sixteen stars, the resulting numbers will be $y = 3.634\,2956$, $x = 58.''1\,32967$. Having therefore reduced the whole sixteen by these mean values of y and x; the error or difference is noted, when compared with the corrected mean refraction which should arise from observation. On a review of these errors, and noticing the mean state of the thermometer for each star, the errors seem to indicate a small correction. I assumed in my former paper, that the refraction as affected by the thermometer, varied in an arithmetical ratio of ,0021 for each degree of FAHRENHEIT's scale; and the mean state to be at 45° for the thermometer without. Continuing the same mean state, and changing the ratio to ,0020, these errors will be affected $\frac{1}{10000}$ of the refraction, for each degree above and below 45°; which being applied, will reduce the final error, as shewn in the last column.

In the same manner I proceeded to find the values of y and x, from the six lower stars, contained in the second table; but the respective values of both y and x were variable, and each

in a decreasing ratio. To discover the law of variation for each, would have been complex; therefore retaining the value of x as general, I found y to vary as the minutes of each degree above 87° × ,0046; the co-efficient (y) of r when so reduced is given; the mean refraction resulting, is contained in the following column; and being corrected for the new factors of the thermometer, there remains the final error.

With a view to assist me in ascertaining whether the refractions were affected by local vapours, Dr. BRINKLEY has kindly communicated to mé some observations of low stars; which when reduced by the same formula do not materially differ from my own. 12 Can. Ven. at 87° 2′ and α Lyræ at 87° 42′ of which there are the greater number of observations, the former gives the same result within half a second, and the latter 1½″.

Several of the fixed Observatories in Europe being situated in sufficiently high latitudes to obtain the elevation of the pole with much correctness; we are thence enabled, by the circumpolar stars, to find the true quantity of refraction, for all zenith distances: and this having been so ascertained, we may apply the same to the observed zenith distance of the sun, at the winter solstice, as a test of its accuracy. With the smaller quantities of refraction, which were used by Dr. BRADLEY and others, fifty years since, it was not possible, that the latitude deduced from the elevation of the pole, and the mean of the solstices, could agree; the distance of the pole from the equator, so computed, would be less than 90°. Hence also, the two solstices would shew an error of double that difference in the obliquity of the ecliptic, when obtained from the greatest and least zenith distances of the sun. The small number of

observations I have been able to make of the sun, to ascertain the agreement of the two solstices, are subjoined; these appear to confirm the refraction, as deduced from the circumpolar stars.

In the course of my observations I have noticed, that applying the correction for the thermometer without, gives the most accurate result. The difference is very sensible in great zenith distances, from the greater quantity of refraction; and we may reasonably infer, that when the front shutter of the Observatory is opened, the horizontal current of the air is of the same temperature as without; although from the short time of the shutter being opened, during the observation, it is not indicated by the thermometer in the telescope. I have therefore constantly used the thermometer without, for the correction, when I have opened the front shutter; but on all other occasions, I have applied the correction for the thermometer within. The instrument is protected from the horizontal current of air, when the sloping shutters in the roof only are opened; the front shutters being five feet above the graduated circle.

Having formerly proposed certain factors for the thermometrical correction of the refraction, and now finding them relatively the same for the thermometer within and without, changing the ratio for each degree of FAHRENHEIT $\frac{1}{10000}$; I shall adopt, hereafter, the following formula. Putting h° for the degree of the scale; then for the thermometer within, $\overline{49^\circ - h^\circ} \times ,0023$ when below the mean; $\overline{49^\circ - h^\circ} \times ,0022$ when above the mean: and for the thermometer without, $\overline{45^\circ - h^\circ} \times 0020$; will produce the respective factors. When h° is less than the mean, these will be positive; when greater

than the mean, negative. These factors of different values will be nearly the same for the thermometer within and without, in summer; when the temperatures approximate: but in winter, when the temperatures may differ 6° or 8°, the factors will vary accordingly. It may therefore not be material, when the quantity of refraction is small, not exceeding $1\frac{1}{2}''$, whether the correction is applied for the thermometer within or without; but when the temperature within is supposed to be affected by the horizontal current from without, I should recommend, in all such cases, the use of the latter correction.

To make a direct comparison of these refractions with the French tables, it may be objected, that the latter being computed for the metrical barometer 0,760 m., which is equal to 29,93 inches of our barometer, the mean refraction from the proposed formula should be increased by the factor +,0111; since we reckon the mean state at 29,60 inches: but, the metrical thermometer at + 10 being equal to ours at 50°; and the mean state being determined from observation, to be 45° of FAHRENHEIT; the factor for 50° will be —,0100; which will nearly compensate the factor for the barometer. I have therefore compared the mean refractions resulting from my formulæ, with those of the French tables, from 80° to 90°: excepting the two last, the difference is not considerable; and whether these arise from defect in the formulæ, or from local causes, can only be determined by observation.

On comparing the refractions, now proposed, with my former deductions, they will not differ one second at 81°. At 79° the present is $\frac{1}{10}$ of a second less than the former; and at 74° 50′ the zenith distance of the winter solstice, 0,″18 must be deducted as a correction. It will appear on inspection of

the second table, that β Persei at 88° 1′ requires but a small equation (— 2″,41) when computed from my former numbers; and at 88° both these formulæ will coincide. I had therefore, in my former paper, determined too hastily, that the formula proposed would agree with observation so far as 88°; not having then discovered the discrepancies between 80° and 88°, which are corrected by the present formula.

In order to facilitate the computation of the true refraction, it is required to form a table of mean refractions from certain formulæ. This may appear difficult at the first view; since every refraction must become equal to, tang. $\overline{z - y^2} \times x$; and unless r is assumed very nearly, several operations may be necessary before the differences will vanish. However, proceeding in small arcs of 10′ to 70°, of 5′ to 86°, of 4′ to 88°, of 3′ to 89°, and of 2′ thence to the horizon, the second differences of the variation of these arcs may always be taken by inspection: and the resulting refraction will be equal to that assumed, in one operation. The correction for the barometer and thermometer will be the sum of the two factors in the annexed tables, into the mean refraction: and the product added thereto, according to the algebraic sign of their sum, will give the true refraction. This method is more expeditious, than by logarithms; no other tables of reference being required: and the computation will be effected with a small number of figures; which is an object I have constantly had in view.

S. GROOMBRIDGE.

Blackheath, 31 January, 1814.

Reduction of the Solstices.

		Zen. dis. corrected.
1810.		° ′ ″
Dec. 14	-	74 · 55 · 53.96
26	-	52.53
27	-	49.75
30	-	52.66
		74 · 55 · 52.23
Nutation	+ 9.60	
Parallax	— 8.59	+ 1.11
Sun's lat.	+ 0.28	
Corr. refr.	— 0.18	
		74 · 55 · 53.34
Latitude		51 . 28 . 2.18
Mean obliq. eclip.		23 . 27 . 51.16

		Zen. dis. corrected.
1811.		° ′ ″
Dec. 9	-	74 · 55 · 53.64
16	-	52.11
22	-	49.79
23	-	55.20
25	-	50.96
1812 30	-	52.30
Jan. 2	-	55.41
		74 · 55 · 52.77
Nutation	+ 9.16	
Parallax	— 8.59	+ 0.22
Sun's lat.	— 0.17	
Corr. refr.	— 0.18	
		74 · 55 · 52.99
Latitude		51 . 28 . 2.18
Mean obliq. eclip.		23 . 27 . 50.81

		Zen. dis. corrected.
1812.		° ′ ″
June 7	-	28 . 0 . 25.76
14	-	23.05
15	-	23.74
29	-	23.04
July 5	-	23.42
		28 . 0 . 23.80
Nutation	— 8.56	
Parallax	— 4.04	— 13.61
Sun's lat.	— 1.01	
		28 . 0 . 10.19
Latitude		51 . 28 . 2.18
Mean obliq. eclip.		23 . 27 . 51.99

		Zen. dis. corrected.
1812.		° ′ ″
Dec. 7	-	74 · 55 · 57.51
8	-	55.51
11	-	55.93
13	-	54.60
		74 · 55 · 54.64
Nutation	+ 7.68	
Parallax	— 8.59	— 1.61
Sun's lat.	— 0.52	
Corr. refr.	— 0.18	
		74 · 55 · 53.03
Latitude		51 . 28 . 2.18
Mean obliq. eclip.		23 . 27 . 50.85

TABLE I.

Star.	No. Obs.	Observed Zenith Distance.	Mean Refraction.	Error from Observ.	Mean Refrac. correc	Mean Therm.	y the coeff. of r.	z the Refrac. at 45°.	Tang. $\frac{z-yr}{x}$ x z.	Error.	Therm. correct.	Final Error.
γ Herculis	15	81 39 30.08	6 19.96	+ 1.06	6 18.90	40	3.62402	58 13601	6 18.84	— 0.06	— 0.22	— 0.28
Capella	30	82 37 49.50	7 6.07	+ 1.12	7 4.95	62	3.55849	58 13114	7 4.49	— 0.46	+ 0.72	+ 0.26
↓ Ursæ maj.	14	82 53 1.76	7 19.80	+ 1.65	7 18.15	40	3.62441	58 13499	7 18.11	— 0.04	— 0.22	— 0.26
β Aurigæ	33	83 29 36.28	7 56.66	+ 2.01	7 54.65	61	3.61064	58 13373	7 54.45	— 0.20	+ 0.76	+ 0.56
ε Cygni	29	83 47 18.26	8 16.55	+ 2.61	8 13.94	40	3.65160	58.13613	8 14.09	+ 0.15	— 0.25	— 0.10
ω Ursæ maj.	10	84 12 4 14	8 47.17	+ 4.08	8 43.09	32	3.74129	58.14097	8 44.21	+ 1.12	— 0.68	+ 0.44
ε Persei	11	84 15 26 81	8 51.62	+ 3.04	8 48.58	53	3.63505	58.13429	8 48.58	0.00	+ 0.42	+ 0.42
λ Ursæ maj.	10	84 31 48.93	9 14.14	+ 4.30	9 9.84	36	3.70809	58.13838	9 10.72	+ 0.88	— 0.50	+ 0.38
ι Aurigæ	13	84 50 58.25	9 42.81	+ 2.64	9 40.17	59	3.54668	58.12771	9 38 85	— 1.32	+ 0.81	— 0.51
11 Lacertæ	15	85 4 10.55	10 4 18	+ 4.37	9 59.81	37	3.63439	58.13276	9 59.79	— 0.02	— 0.48	— 0.50
κ Andromedæ	11	85 4 32.79	10 4.79	+ 4 48	10 0.31	42	3.64000	58.13328	10 0.40	+ 0.09	— 0.18	— 0.09
ξ Cygni	14	85 10 51 26	10 15.52	+ 4 83	10 10.69	39	3.64778	58.13364	10 10.91	+ 0.22	— 0 37	— 0.15
μ Ursæ maj.	12	85 53 46.42	11 38 53	+ 7.02	11 31 51	36	3.65335	58.13204	11 31.95	+ 0.44	— 0.62	— 0.18
ι Andromedæ	6	86 6 22.45	12 6.71	+ 7.41	11 59.30	43	3.63706	58.13029	11 59.38	+ 0.08	— 0.14	— 0.06
γ Andromedæ	14	86 53 26.64	14 11.52	+ 10.42	14 1.10	55	3.61333	58.12594	14 0.29	— 0.81	+ 0.84	+ 0.03
ο Andromedæ	6	86 58 29.91	14 27 05	+ 11.32	14 15.73	435	3.62255	58.12617	14 15.27	— 0.46	— 0.13	— 0 59

TABLE II.

β Bootis	10	87 8 19.70	14 58.62	+ 12.30	14 46.32	34	3.59582		14 47.49	+ 1.17	— 0.98	+ 0.19
α Aurigæ	11	87 19 9 81	15 35 64	+ 7 24	15 28.40	60	3.54576		15 25.91	— 2.49	+ 1.40	— 1.09
ζ Aurigæ	15	87 29 7 86	16 11.89	+ 6.18	16 5.71	57	3.49971		16 3.85	— 1.86	+ 1.16	— 0.70
β Persei	16	88 1 0.03	18 23.73	— 2.41	18 26 14	54	3.35247		18 24.77	— 1.37	+ 1.00	— 0.37
γ Cygni	5	88 31 1.12	20 53.78	—16 0	21 9 78	39	3.21379		21 11.37	+ 1.59	— 0.76	+ 0.83
ε Persei	5	88 42 35 53	21 59.32	—28.78	22 28.10	53	3.16032		22 26.48	— 1.62	+ 1.08	— 0.54

58° 132967

Mean Refraction.

Zen. dis.	French tables.	S. G.	Value of y.
°	′ ″	′ ″	
80	5 19.8	5 19.18	3.63429
81	5 53.5	5 52.83	——
82	6 34.4	6 33.79	——
83	7 24.7	7 24.03	——
84	8 29.9	8 29.13	——
85	9 54.3	9 53.03	——
86	11 48.3	11 45.27	——
87	14 28.1	14 19.80	3.63429
88	18 22.2	18 19.83	3.35709
89	24 21.2	24 32.94	3.07989
90	33 46.3	34 28.13	2.80269

The two formulæ compared.

Zen. dis.	3.36257 × 58″.1192	3.634297 × 58″.132967
°	′ ″	′ ″
70	2 38.41	2 38.34
71	2 47.31	2 47.23
72	2 57.13	2 57.03
73	3 8.04	3 7.92
74	3 20.22	3 20.07
75	3 33.92	3 33.73
76	3 49.44	3 49.21
77	4 7.19	4 6.89
78	4 27.68	4 27.30
79	4 51.59	4 51.09
80	5 19.85	5 19.18
81	5 53.74	5 52.83
82	6 35.06	6 33.79
83	7 26.46	7 24.03
84	8 31.85	8 29.13
85	9 57.27	9 53.03
86	11 52.21	11 45.27
87	14 31.75	14 19.80
88	18 19.19	18 19.83

Mean Refraction calculated at 58″,132967 for 45 Degrees of apparent Zenith Distance × Tang. Z

Zen. Dis.	Refrac.	Diff.
° ′	′ ″	″
0 0	0 0.00	0.17
10	0 0.17	0.17
20	0 0.34	0.17
30	0 0.51	0.17
40	0 0.68	0.16
50	0 0.84	0.17
1 0	0 1.01	0.17
10	0 1.18	0.17
20	0 1.35	0.17
30	0 1.52	0.17
40	0 1.69	0.17
50	0 1.86	0.17
2 0	0 2.03	0.17
10	0 2.20	0.17
20	0 2.37	0.16
30	0 2.53	0.17
40	0 2.70	0.17
50	0 2.87	0.17
3 0	0 3.04	0.17
10	0 3.21	0.17
20	0 3.38	0.17
30	0 3.55	0.17
40	0 3.72	0.17
50	0 3.89	0.17
4 0	0 4.06	0.17
10	0 4.23	0.17
20	0 4.40	0.17
30	0 4.57	0.17
40	0 4.74	0.17
50	0 4.91	0.17
5 0	0 5.08	0.17
10	0 5.25	0.17
20	0 5.42	0.17
30	0 5.59	0.17
40	0 5.76	0.17
50	0 5.93	0.17
6 0	0 6.10	0.17
10	0 6.27	0.17
20	0 6.44	0.18
30	0 6.62	0.17
40	0 6.79	0.17
50	0 6.96	0.17
7 0	0 7.13	0.17
10	0 7.30	0.17
20	0 7.47	0.17
30	0 7.64	0.18
40	0 7.82	0.17
50	0 7.99	0.17
8 0	0 8.16	0.17
10	0 8.33	0.17
20	0 8.50	0.18
30	0 8.68	0.17
40	0 8.85	0.17
50	0 9.02	0.18
9 0	0 9.20	0.17
10	0 9.37	0.17
20	0 9.54	0.18
30	0 9.72	0.17
40	0 9.89	0.17
50	0 10.06	0.18
10 0	0 10.24	0.17
10	0 10.41	0.18
20	0 10.59	0.17
30	0 10.76	0.18
40	0 10.94	0.17
50	0 11.11	0.18
11 0	0 11.29	0.17
10	0 11.46	0.18
20	0 11.64	0.17
30	0 11.81	0.18
40	0 11.99	0.18
50	0 12.17	0.17
12 0	0 12.34	0.18
10	0 12.52	0.18
20	0 12.70	0.17
30	0 12.87	0.18
40	0 13.05	0.17
50	0 13.22	0.18
13 0	0 13.40	0.18
10	0 13.58	0.18
20	0 13.76	0.18
30	0 13.94	0.18
40	0 14.12	0.18
50	0 14.30	0.18
14 0	0 14.48	0.18
10	0 14.66	0.18
20	0 14.84	0.18
30	0 15.02	0.18
40	0 15.20	0.18
50	0 15.38	0.18
15 0	0 15.56	0.18
10	0 15.74	0.18
20	0 15.92	0.18
30	0 16.10	0.18
40	0 16.28	0.19
50	0 16.47	0.18
16 0	0 16.65	0.18
10	0 16.83	0.18
20	0 17.01	0.19
30	0 17.20	0.18
40	0 17.38	0.19
50	0 17.57	0.18
17 0	0 17.75	0.19
10	0 17.94	0.18
20	0 18.12	0.19
30	0 18.31	0.18
40	0 18.49	0.19
50	0 18.68	0.18
18 0	0 18.86	0.19
10	0 19.05	0.19
20	0 19.24	0.19
30	0 19.43	0.18
40	0 19.61	0.19
50	0 19.80	0.19
19 0	0 19.99	0.19
10	0 20.18	0.19
20	0 20.37	0.19
30	0 20.56	0.19
40	0 20.75	0.19
50	0 20.94	0.19
20 0	0 21.13	0.19
10	0 21.32	0.19
20	0 21.51	0.20
30	0 21.71	0.19
40	0 21.90	0.19
50	0 22.09	0.20
21 0	0 22.29	0.19
10	0 22.48	0.19
20	0 22.67	0.20
30	0 22.87	0.19
40	0 23.06	0.20
50	0 23.26	0.20
22 0	0 23.46	0.19
10	0 23.65	0.20
20	0 23.85	0.20
30	0 24.05	0.20
40	0 24.25	0.20
50	0 24.45	0.20
23 0	0 24.65	0.20
10	0 24.85	0.20
20	0 25.05	0.20
30	0 25.25	0.20
40	0 25.45	0.20
50	0 25.65	0.20
24 0	0 25.85	0.20
10	0 26.05	0.21
20	0 26.26	0.20
30	0 26.46	0.20
40	0 26.66	0.21
50	0 26.87	0.20
25 0	0 27.07	0.21
10	C 27 28	0.21
20	0 27.49	0.20
30	0 27.69	0.21
40	0 27.90	0.21
50	0 28.11	0.21
26 0	0 28.32	0.21
10	0 28.53	0.21
20	0 28.74	0.21
30	0 28.95	0.21
40	0 29.16	0.21
50	0 29.37	0.21
27 0	0 29.58	0.22
10	0 29.80	0.20
20	0 30.00	0.22
30	0 30.22	0.22
40	0 30.44	0.21
50	0 30.65	0.22
28 0	0 30.87	0.22
10	0 31.09	0.21
20	0 31.30	0.22
30	0 31.52	0.22
40	0 31.74	0.22
50	0 31.96	0.22
29 0	0 32.18	0.22
10	0 32.40	0.22
20	0 32.62	0.23
30	0 32.85	0.22
40	0 33.07	0.22
50	0 33.29	0.23
30 0	0 33.52	0.22
10	0 33.74	0.23
20	0 33.97	0.23
30	0 34.20	0.22
40	0 34.42	0.23
50	0 34.65	0.23
31 0	0 34.88	0.23
10	0 35.11	0.23
20	0 35.34	0.23
30	0 35.57	0.24
40	0 35.81	0.23
50	0 36.04	0.23
32 0	0 36.27	0.24
10	0 36.51	0.23
20	0 36.74	0.24
30	0 36.98	0.24
40	0 37.22	0.24
50	0 37.46	0.24
33 0	0 37.70	0.24
10	0 37.94	0.24
20	0 38.18	0.24
30	0 38.42	0.24
40	0 38.66	0.25
50	0 38.91	0.24
34 0	0 39.15	0.25
10	0 39.40	0.24
20	0 39.64	0.25
30	0 39.89	0.25
40	0 40.14	0.25
50	0 40.39	0.25
35 0	0 40.64	0.25
10	0 40.89	0.26
20	0 41.15	0.25
30	0 41.40	0.26
40	0 41.66	0.25
50	0 41.91	0.26
36 0	0 42.17	0.26
10	0 42.43	0.26
20	0 42.69	0.26
30	0 42.95	0.26
40	0 43.21	0.26
50	0 43.47	0.27
37 0	0 43.74	0.26
10	0 44.00	0.27
20	0 44.27	0.26
30	0 44.53	0.27
40	0 44.80	0.27
50	0 45.07	0.27
38 0	0 45.34	0.28
10	0 45.62	0.27
20	0 45.89	0.27
30	0 46.16	0.28
40	0 46.44	0.28
50	0 46.72	0.28
39 0	0 47.00	0.28
10	0 47.28	0.28
20	0 47.56	0.28
30	0 47.84	0.28
40	0 48.12	0.29
50	0 48.41	0.28
40 0	0 48.69	0.29
10	0 48.98	0.29
20	0 49.27	0.29
30	0 49.56	0.29
40	0 49.85	0.30
50	0 50.15	0.29
41 0	0 50.44	0.30
10	0 50.74	0.30
20	0 51.04	0.30
30	0 51.34	0.30
40	0 51.64	0.30
50	0 51.94	0.31
42 0	0 52.25	0.30
10	0 52.55	0.31
20	0 52.86	0.31
30	0 53.17	0.31
40	0 53.48	0.31
50	0 53.79	0.32
43 0	0 54.11	0.31
10	0 54.42	0.32
20	0 54.74	0.32
30	0 55.06	0.32
40	0 55.38	0.32
50	0 55.70	0.32
44 0	0 56.03	0.32
10	0 56.35	0.33
20	0 56.68	0.33
30	0 57.01	0.33
40	0 57.34	0.34
50	0 57.68	0.33
45 0	0 58.01	0.33
10	0 58.35	0.34
20	0 58.69	0.34
30	0 59.03	0.35
40	0 59.38	0.34
50	0 59.72	0.35
46 0	1 0.07	0.35
10	1 0.42	0.35
20	1 0.77	0.36
30	1 1.13	0.35
40	1 1.48	0.36
50	1 1.84	0.36
47 0	1 2.20	0.37
10	1 2.57	0.36
20	1 2.93	0.37
30	1 3.30	0.37
40	1 3.67	0.37
50	1 4.04	0.37
48 0	1 4.41	0.38
10	1 4.79	0.38
20	1 5.17	0.38
30	1 5.55	0.39
40	1 5.94	0.39
50	1 6.33	0.39
49 0	1 6.72	0.39
10	1 7.11	0.39
20	1 7.50	0.40
30	1 7.90	0.40
40	1 8.30	0.40
50	1 8.70	0.41
50 0	1 9.11	0.41
10	1 9.52	0.41
20	1 9.93	0.41
30	1 10.34	0.42
40	1 10.76	0.42
50	1 11.18	0.42
51 0	1 11.60	0.43
10	1 12.03	0.43
20	1 12.46	0.43
30	1 12.89	0.44
40	1 13.33	0.44
50	1 13.77	0.44
52 0	1 14.21	0.44
10	1 14.65	0.45
20	1 15.10	0.45
30	1 15.55	0.46
40	1 16.01	0.46
50	1 16.47	0.46
53 0	1 16.93	0.46
10	1 17.39	0.47
20	1 17.86	0.47
30	1 18.33	0.48
40	1 18.81	0.48
50	1 19.29	0.49
54 0	1 19.78	0.4
10	1 20.27	0.4
20	1 20.76	0.4
30	1 21.25	0.5
40	1 21.75	0.5
50	1 22.26	0.5
55 0	1 22.77	0.
10	1 23.28	0.
20	1 23.79	0.
30	1 24.31	0.
40	1 24.84	0.
50	1 25.37	0.
56 0	1 25.91	0.
10	1 26.45	0.
20	1 26.99	0.
30	1 27.54	0.
40	1 28.09	0.
50	1 28.65	0.
57 0	1 29.21	0.
10	1 29.78	0.
20	1 30.35	0.
30	1 30.93	0.
40	1 31.51	0.
50	1 32.10	0.
58 0	1 32.69	0.
10	1 33.29	0.
20	1 33.90	0.
30	1 34.51	0.
40	1 35.13	0.
50	1 35.75	0.
59 0	1 36.38	0.
10	1 37.01	0.
20	1 37.65	0.
30	1 38.30	0.
40	1 38.95	0.
50	1 39.61	0.
60 0	1 40.28	0.
10	1 40.95	0.
20	1 41.63	0.
30	1 42.32	0.
40	1 43.01	0.
50	1 43.71	0.
61 0	1 44.42	0.
10	1 45.14	0.
20	1 45.86	0.
30	1 46.59	0.
40	1 47.33	0.
50	1 48.08	0.
62 0	1 48.83	0.
10	1 49.59	0.
20	1 50.36	0.
30	1 51.14	0.
40	1 51.93	0.
50	1 52.73	0.

ng. Z — 3.6342956r so far as 87° Zen. Dis. below which r is reduced, 00462 for each minute.

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Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.	Zen. Dis.	Refrac.	Diff.
° ′	° ′ ″	″	° ′	° ′ ″	″	° ′	° ′ ″	″	° ′	° ′ ″	″	° ′	° ′ ″	″	° ′	° ′ ″	″
0 0	0 0 0	0.00	63 0	1 53.53	0.82	71 0	2 47.23	0.78	75 30	3 41.22	1.30	80 0	5 19.18	2.57	84 30	9 8.18	7.05	88 18	19 54.47	17.06
10	0 49	0.49	10	1 54.35	0.82	5	2 48.01	0.79	35	3 42.52	1.31	5	5 21.75	2.61	35	9 15.23	7.20	21	20 11.53	17.45
20	0 49	0.49	20	1 55.17	0.83	10	2 48.80	0.79	40	3 43.83	1.32	10	5 24.36	2.65	40	9 22.43	7.38	24	20 28.98	17.84
30	0 49	0.49	30	1 56.00	0.83	15	2 49.59	0.79	45	3 45.15	1.34	15	5 27.01	2.70	45	9 29.81	7.56	27	20 46.82	18.20
40	0 51	0.51	40	1 56.85	0.85	20	2 50.39	0.81	50	3 46.49	1.35	20	5 29.71	2.73	50	9 37.37	7.73	30	21 5.08	18.68
50	0 51	0.51	50	1 57.70	0.86	25	2 51.20	0.81	55	3 47.84	1.37	25	5 32.44	2.77	55	9 45.10	7.93	33	21 23.76	19.12
1 0	0 52	0.52	64 0	1 58.56	0.87	30	2 52.01	0.82	76 0	3 49.21	1.38	30	5 35.21	2.82	85 0	9 53.03	8.12	36	21 42.88	19.57
10	0 52	0.52	10	1 59.43	0.89	35	2 52.83	0.82	5	3 50.59	1.40	35	5 38.03	2.87	5	10 1.15	8.31	39	22 2.45	20.02
20	0 53	0.53	20	2 0.32	0.89	40	2 53.65	0.84	10	3 51.99	1.42	40	5 40.90	2.91	10	10 9.46	8.53	42	22 22.47	20.49
30	0 53	0.53	30	2 1.21	0.90	45	2 54.49	0.84	15	3 53.41	1.43	45	5 43.81	2.96	15	10 17.99	8.74	45	22 42.96	20.99
40	0 54	0.54	40	2 2.12	0.91	50	2 55.33	0.85	20	3 54.84	1.45	50	5 46.77	3.01	20	10 26.73	8.96	48	23 3.95	21.47
50	0 54	0.54	50	2 3.03	0.93	55	2 56.18	0.85	25	3 56.29	1.46	55	5 49.78	3.05	25	10 35.69	9.19	51	23 25.42	21.98
2 0	0 55	0.55	65 0	2 3.96	0.94	72 0	2 57.03	0.86	30	3 57.75	1.48	81 0	5 52.83	3.10	30	10 44.88	9.43	54	23 47.40	22.50
10	0 56	0.56	10	2 4.90	0.95	5	2 57.89	0.86	35	3 59.23	1.50	5	5 55.93	3.16	35	10 54.31	9.67	57	24 9.90	23.04
20	0 56	0.56	20	2 5.85	0.96	10	2 58.76	0.87	40	4 0.73	1.52	10	5 59.09	3.21	40	11 3.98	9.92	89 0	24 32.94	15.66
30	0 57	0.57	30	2 6.81	0.97	15	2 59.64	0.88	45	4 2.25	1.53	15	6 2.30	3.26	45	11 13.90	10.19	2	24 48.60	15.91
40	0 58	0.58	40	2 7.78	0.97	20	3 0.53	0.90	50	4 3.78	1.53	20	6 5.56	3.32	50	11 24.09	10.45	4	25 4.51	16.16
50	0 59	0.59	50	2 8.77	1.00	25	3 1.43	0.90	55	4 5.33	1.56	25	6 8.88	3.38	55	11 34.54	10.73	6	25 20.67	16.42
3 0	1 0	0.60	66 0	2 9.77	1.01	30	3 2.33	0.91	77 0	4 6.89	1.59	30	6 12.26	3.43	86 0	11 45.27	8.80	8	25 37.09	16.68
10	1 0	0.60	10	2 10.78	1.03	35	3 3.24	0.92	5	4 8.48	1.61	35	6 15.69	3.50	4	11 54.07	8.99	10	25 53.77	16.94
20	1 0	0.61	20	2 11.81	1.04	40	3 4.16	0.93	10	4 10.09	1.63	40	6 19.19	3.55	8	12 3.06	9.18	12	26 10.71	17.20
30	1 0	0.62	30	2 12.85	1.05	45	3 5.09	0.93	15	4 11.72	1.65	45	6 22.74	3.62	12	12 12.24	9.39	14	26 27.91	17.49
40	1 0	0.62	40	2 13.90	1.07	50	3 6.02	0.93	20	4 13.37	1.66	50	6 26.36	3.68	16	12 21.63	9.59	16	26 45.40	17.76
50	1 0	0.63	50	2 14.97	1.08	55	3 6.97	0.95	25	4 15.03	1.69	55	6 30.04	3.75	20	12 31.22	9.80	18	27 3.16	18.04
4 0	1 0	0.63	67 0	2 16.05	1.09	73 0	3 7.92	0.96	30	4 16.72	1.71	82 0	6 33.79	3.82	24	12 41.02	10.02	20	27 21.20	18.32
10	1 0	0.64	10	2 17.14	1.11	5	3 8.88	0.97	35	4 18.43	1.73	5	6 37.61	3.89	28	12 51.04	10.24	22	27 39.52	18.62
20	1 0	0.65	20	2 18.25	1.13	10	3 9.85	0.98	40	4 20.16	1.75	10	6 41.50	3.96	32	13 1.28	10.47	24	27 58.14	18.91
30	1 0	0.65	30	2 19.38	1.14	15	3 10.83	0.99	45	4 21.91	1.77	15	6 45.46	4.03	36	13 11.75	10.71	26	28 17.05	19.21
40	1 0	0.66	40	2 20.52	1.16	20	3 11.82	1.00	50	4 23.68	1.80	20	6 49.49	4.10	40	13 22.46	10.96	28	28 36.26	19.52
50	1 0	0.67	50	2 21.68	1.17	25	3 12.82	1.00	55	4 25.48	1.82	25	6 53.59	4.19	44	13 33.42	11.20	30	28 55.78	19.82
5 0	1 0	0.68	68 0	2 22.85	1.19	30	3 13.82	1.02	78 0	4 27.30	1.84	30	6 57.78	4.26	48	13 44.62	11.46	32	29 15.60	20.14
10	1 0	0.68	10	2 24.04	1.21	35	3 14.84	1.02	5	4 29.14	1.87	35	7 2.04	4.35	52	13 56.08	11.73	34	29 35.74	20.45
20	1 0	0.69	20	2 25.25	1.22	40	3 15.86	1.04	10	4 31.01	1.89	40	7 6.39	4.43	56	14 7.81	11.99	36	29 56.19	20.78
30	1 0	0.70	30	2 26.47	1.24	45	3 16.90	1.05	15	4 32.90	1.91	45	7 10.82	4.51	87 0	14 19.80	13.11	38	30 16.97	21.10
40	1 0	0.70	40	2 27.71	1.26	50	3 17.95	1.06	20	4 34.81	1.94	50	7 15.33	4.60	4	14 32.91	13.46	40	30 38.07	21.44
50	1 0	0.71	50	2 28.97	1.28	55	3 19.01	1.06	25	4 36.75	1.97	55	7 19.93	4.70	8	14 46.37	13.83	42	30 59.51	21.77
6 0	1 0	0.72	69 0	2 30.25	1.30	74 0	3 20.07	1.08	30	4 38.72	1.99	83 0	7 24.63	4.79	12	15 0.20	14.20	44	31 21.28	22.11
10	1 0	0.73	10	2 31.55	1.32	5	3 21.15	1.08	35	4 40.71	2.02	5	7 29.42	4.88	16	15 14.40	14.60	46	31 43.39	22.46
20	1 0	0.73	20	2 32.87	1.34	10	3 22.23	1.10	40	4 42.73	2.05	10	7 34.30	4.99	20	15 29.00	15.00	48	32 5.85	22.81
30	1 0	0.74	30	2 34.21	1.35	15	3 23.33	1.11	45	4 44.78	2.07	15	7 39.29	5.09	24	15 44.00	15.42	50	32 28.66	23.16
40	1 0	0.75	40	2 35.56	1.38	20	3 24.44	1.12	50	4 46.85	2.11	20	7 44.38	5.19	28	15 59.42	15.85	52	32 51.82	23.53
50	1 0	0.75	50	2 36.94	1.40	25	3 25.56	1.13	55	4 48.96	2.13	25	7 49.57	5.30	32	16 15.27	16.31	54	33 15.35	23.89
7 0	1 0	0.76	70 0	2 38.34	1.41	30	3 26.69	1.15	79 0	4 51.09	2.16	30	7 54.87	5.41	36	16 31.58	16.77	56	33 39.24	24.26
10	1 0	0.77	5	2 39.05	1.42	35	3 27.84	1.15	5	4 53.25	2.19	35	8 0.28	5.52	40	16 48.35	17.26	58	34 3.50	24.63
20	1 0	0.78	10	2 39.76	1.43	40	3 28.99	1.17	10	4 55.44	2.23	40	8 5.80	5.65	44	17 5.01	17.76	90 0	34 28.13	25.02
20	1 0	0.79	15	2 40.48	1.44	45	3 30.16	1.18	15	4 57.67	2.25	45	8 11.45	5.76	48	17 23.37	18.27	2	34 53.15	25.40
30	1 0	0.80	20	2 41.21	1.45	50	3 31.34	1.19	20	4 59.92	2.29	50	8 17.21	5.89	52	17 41.64	18.82	4	35 18.55	25.79
40	1 0	0.81	25	2 41.94	1.46	55	3 32.53	1.20	25	5 2.21	2.32	55	8 23.10	6.03	56	18 0.46	19.37	6	35 44.34	26.18
50	1 0	0.82	30	2 42.68	1.47	75 0	3 33.73	1.22	30	5 4.53	2.35	84 0	8 29.13	6.15	88 0	18 19.83	14.91	8	36 10.52	26.58
8 0	1 0	0.83	35	2 43.42	1.47	5	3 34.95	1.23	35	5 6.88	2.39	5	8 35.28	6.29	3	18 34.74	15.24	10	36 37.10	26.98
10	1 0	0.84	40	2 44.17	1.48	10	3 36.18	1.24	40	5 9.27	2.42	10	8 41.57	6.43	6	18 49.98	15.59	12	37 4.08	27.39
20	1 0	0.85	45	2 44.93	1.49	15	3 37.42	1.25	45	5 11.69	2.46	15	8 48.00	6.57	9	19 5.57	15.94	14	37 31.47	27.81
30	1 0	0.86	50	2 45.69	1.50	20	3 38.67	1.27	50	5 14.15	2.50	20	8 54.57	6.73	12	19 21.51	16.29	16	37 59.28	28.22
40	1 0	0.87	55	2 46.46	1.50	25	3 39.94	1.28	55	5 16.65	2.53	25	9 1.30	6.88	15	19 37.80	16.67	18	38 27.50	
50	1 0	0.87																		

Factors for the correction of the mean Refraction.

Barometer in inches.				Fahrenheit's Thermometer.							
	−		+	°	Within. +	Without. +	°	Within. −	Without. −
28.60	,0350	29.60	,0000	21.5	,0632	,0470	49.0	,0000	,0080
62	,0342	62	,0007	22.0	,0621	,0460	49.5	,0011	,0090
64	,0335	64	,0014	22.5	,0609	,0450	50.0	,0022	,0100
66	,0328	66	,0020	23.0	,0598	,0440	50.5	,0033	,0110
68	,0321	68	,0027	23.5	,0586	,0430	51.0	,0044	,0120
28.70	,0314	29.70	,0034	24.0	,0575	,0420	51.5	,0055	,0130
72	,0306	72	,0041	24.5	,0563	,0410	52.0	,0066	,0140
74	,0299	74	,0047	25.0	,0552	,0400	52.5	,0077	,0150
76	,0292	76	,0054	25.5	,0540	,0390	53.0	,0088	,0160
78	,0285	78	,0061	26.0	,0529	,0380	53.5	,0099	,0170
28.80	,0278	29.80	,0068	26.5	,0517	,0370	54.0	,0110	,0180
82	,0271	82	,0074	27.0	,0506	,0360	54.5	,0121	,0190
84	,0264	84	,0081	27.5	,0494	,0350	55.0	,0132	,0200
86	,0256	86	,0088	28.0	,0483	,0340	55.5	,0143	,0210
88	,0249	88	,0095	28.5	,0471	,0330	56.0	,0154	,0220
28.90	,0242	29.90	,0101	29.0	,0460	,0320	56.5	,0165	,0230
92	,0235	92	,0108	29.5	,0448	,0310	57.0	,0176	,0240
94	,0228	94	,0115	30.0	,0437	,0300	57.5	,0187	,0250
96	,0221	96	,0122	30.5	,0425	,0290	58.0	,0198	,0260
98	,0214	98	,0128	31.0	,0414	,0280	58.5	,0209	,0270
29.00	,0207	30.00	,0135	31.5	,0402	,0270	59.0	,0220	,0280
02	,0200	02	,0142	32.0	,0391	,0260	59.5	,0231	,0290
04	,0193	04	,0149	32.5	,0379	,0250	60.0	,0242	,0300
06	,0186	06	,0155	33.0	,0368	,0240	60.5	,0253	,0310
08	,0179	08	,0162	33.5	,0356	,0230	61.0	,0264	,0320
29.10	,0172	30.10	,0169	34.0	,0345	,0220	61.5	,0275	,0330
12	,0165	12	,0176	34.5	,0333	,0210	62.0	,0286	,0340
14	,0158	14	,0182	35.0	,0322	,0200	62.5	,0297	,0350
16	,0151	16	,0189	35.5	,0310	,0190	63.0	,0308	,0360
18	,0144	18	,0196	36.0	,0299	,0180	63.5	,0319	,0370
29.20	,0137	30.20	,0203	36.5	,0287	,0170	64.0	,0330	,0380
22	,0130	22	,0210	37.0	,0276	,0160	64.5	,0341	,0390
24	,0123	24	,0216	37.5	,0264	,0150	65.0	,0352	,0400
26	,0116	26	,0223	38.0	,0253	,0140	65.5	,0363	,0410
28	,0109	28	,0230	38.5	,0241	,0130	66.0	,0374	,0420
29.30	,0102	30.30	,0237	39.0	,0230	,0120	66.5	,0385	,0430
32	,0096	32	,0243	39.5	,0218	,0110	67.0	,0396	,0440
34	,0089	34	,0250	40.0	,0207	,0100	67.5	,0407	,0450
36	,0082	36	,025	40.5	,0195	,0090	68.0	,0418	,0460
38	,0075	38	,0264	41.0	,0184	,0080	68.5	,0429	,0470
29.40	,0068	30.40	,027	41.5	,0172	,0070	69.0	,0440	,0480
42	,0061	42	,0277	42.0	,0161	,0060	69.5	,0451	,0490
44	,0054	44	,0284	42.5	,0149	,0050	70.0	,0462	,0500
46	,0048	46	,0291	43.0	,0138	,0040	70.5	,0473	,0510
48	,0041	48	,0297	43.5	,0126	,0030	71.0	,0484	,0520
29.50	,0034	30.50	,0304	44.0	,0115	,0020	71.5	,0495	,0530
52	,0027	52	,0311	44.5	,0103	,0010	72.0	,0506	,0540
54	,0020	54	,0318	45.0	,0092	—	72.5	,0517	,0550
56	,0014	56	,0324	45.5	,0080	,0010	73.0	,0528	,0560
58	,0007	58	,0331	46.0	,0069	,0020	73.5	,0539	,0570
				46.5	,0057	,0030	74.0	,0550	,0580
				47.0	,0046	,0040	74.5	,0561	,0590
				47.5	,0034	,0050	75.0	,0572	,0600
				48.0	,0023	,0060	75.5	,0583	,0610
				48.5	,0011	,0070	76.0	,0594	,0620

XVIII. *Propositions containing some Properties of* TANGENTS *to Circles; and of* TRAPEZIUMS *inscribed in Circles, and non-inscribed. Together with Propositions on the* Elliptic *Representations of* CIRCLES, *upon a plane surface, by* PERSPECTIVE. *By* Richard Hey, *LL. D.; late Fellow of Sidney Sussex and Magdalen Colleges, in the University of Cambridge. Communicated by the Rev.* Edward Balme, *M. A. F. R. S.*

Read March 31, 1814.

THE propositions of the first series, and which are termed Introductory, had their origin from my attention being directed to the perspective representation of a Circle. But they are kept separate from the perspective propositions. Comprising some things believed to be new, and forming a short system in themselves, they may, on these accounts, be acceptable to other Readers besides those who study perspective.

A desideratum, in the science of perspective, had been suggested to me by the Reverend Mr. KERRICH, Principal Librarian to the University of Cambridge; namely, of some *law* or *laws* which the axes, of ellipses representing circles, might be found to observe in their directions. An attempt, grounded on this suggestion, is contained in the four Propositions concluding the second or Perspective series. If these be correct, it is hoped that a foundation is laid, for further improvement by persons younger than myself and differently circumstanced. And some mechanical instrument * may

* The invention of an Instrument for the ready delineation of Ellipses, by Mr.

possibly be brought to co-operate with the theory here given or begun, so as that, in conjunction, they may afford considerable advantage to the practice of perspective; especially where a number of circles together are to be represented.

The same four propositions are built upon the sixth on perspective: of which the construction (as given in fig. 5.) may be seen in HAMILTON's * Stereography. That it was not taken from his work, but separately discovered, has no claim to the attention of the reader, except in this respect; that, in the solutions of any problem, the more persons there are who give independently the same solution, the more likely it is (cæteris paribus) to be better than any solution not yet discovered. The *demonstration* also has been varied; both of this sixth proposition, and of some other things which, though not new in substance, it has been judged expedient to insert.

For the seventh of the propositions on perspective, I am also indebted to Mr. KERRICH: though, for the proof here given of it, I am responsible.

The foregoing account appears to be all the previous information necessary to trouble a reader with, who may be going to examine the propositions in his closet. But, as mathematical demonstrations, accompanied with diagrams, cannot be accommodated to a public reading, some further particulars are therefore here subjoined: which, if the Paper should have the honour of being received by the Royal Society, may give a better insight into its nature and intent.

The whole had its origin in the study of that part, of the

John Farey junior, obtained the gold medal, in May 1813, from the Society for the Encouragement of Arts. For its merits I refer to that Society.

* B. III. Sect. II. Prob. 1.

science of perspective, which relates to the representation of a *circle*. And the principal result (so far as immediately respects that science) is contained in the four propositions which conclude the paper, together with the lemma which precedes them. This is called a lemma rather than a proposition, because it does not necessarily involve the consideration of a circle.

That the perspective representation of an entire circle is an ellipse or circle, is simply a part of the doctrine of conic sections. To that doctrine therefore it is referred, in the first of the perspective propositions. An *entire* circle, supposed to be viewed by the eye through the plane of the picture, excludes (by the hypothesis) the two cases in which the circle touches and cuts the plane, passing through the eye, parallel to the picture. In which two cases, the representation is a parabola and hyperbola respectively. Not having any thing new to offer upon these representations, or not any thing appearing likely to improve the practice of perspective, I have omitted the two cases entirely; and confined myself to the *elliptic* representations, the circular included.

The principal inquiry has been, whether, in any instances of a number of circles, lying in one plane and having their centres in one right line, any *law* or *laws* could be discovered to be observed by the major or minor axes of the representing ellipses, in their directions or positions. If it could have been proved that the axes converged to a known or easily discoverable point, this would have been a great acquisition to the science of perspective, and have given a direct facility to the practice. But, if indeed they do *not* so converge, it is natural to wish that this should be demonstrated. Such a

demonstration puts an end to further inquiry about the point to which they might be imagined to converge, and wholly about such convergency in them; and leaves us free to inquire after any other law or regularity which the axes *may* observe.

Now the object of the lemma (with its first corollary), prefixed to the eighth of the propositions on perspective, is to prove that lines, in a picture, cannot converge to a point; unless the original lines which they represent happen so to converge or else to be parallel. The inference is, that the axes of the ellipses cannot so converge, unless the chords which they represent should so converge or be parallel: a case which may rarely happen, and, if happening, may not be readily discernible.

The convergency to a point, by the axes, being therefore dismissed, the four remaining propositions, after the lemma, exhibit nevertheless a regularity in the directions of the axes. Though these do not converge to a point, they are (if the reasoning be just) parallel respectively to other lines which *do* converge to a point. To find each of these other lines separately and accurately, is not the object proposed. But certain *limits* are pointed out, within which the artist is to keep; and a regularity marked, by which the variation in the directions of successive axes is to be guided: which, it is hoped, will produce a sufficient accuracy in a great number of instances. And, where any one or more circles require singly a strict accuracy, to this the construction of the sixth proposition is adapted.

How much, or whether any part, of the first seven propositions on perspective, might have been omitted, without injury

to that series of propositions, I do not take upon me to decide. Certainly much of the matter, contained in the seven, is not new. But, so far as it is necessary to the principal design, its admission will be justified. And also new *proof*, of what is already known, is sometimes acceptable to the mathematician, and may be admitted as an advancement of science. On which ground the proof here given of the sixth proposition, and some other proofs, await the decision of the reader.

An indirect result of the research, is merely geometrical and may be taken separately from perspective. This consists of the introductory propositions. On the one hand, it appeared necessary, or desirable, that these should accompany the perspective propositions; because of various references, in the latter, to the former. On the other hand, it seemed also desirable that the introductory propositions should be so far kept separate, as to enable any reader to peruse them, who would have declined all examination of their contents if interwoven with perspective.

The properties of the circle, not improbably, lie open to abundant successful investigation. It is hoped that the introductory propositions, of this paper, may have contributed a little to the stock of knowledge respecting that remarkable geometrical figure.

To avoid crowding the plates with figures, I have, in a few instances, left it to the reader to construct his own figures for illustration, if they appear to be wanted. For instance, in cor. 6 to introd. prop. X, and cor. 1 to introd. prop. XII.

INTRODUCTORY PROPOSITIONS.

Some Properties of TANGENTS *to Circles; and of* TRAPEZIUMS, *inscribed in Circles, and non-inscribed.*

Definitions, for the Introductory Propositions.

Def. 1. The *tangent-chord*, of a given point without a circle, is the chord from whose extremities tangents meet in the given point.

Def. 2. A *vertex* to a base is any point, within a circle, through which are drawn chords: and the right line joining the points to which those are the tangent-chords, is the *base* to that vertex.

Def. 3. The *mean point* is a point in the diameter perpendicular to such base: and its distance from the base is a mean proportional between the distances of the extremities of that diameter, from the base.

Thus (Fig. 1): If the chords PQ, GH, pass through V within a circle, and if PD, QD, GL, HL, be tangents, and LD be drawn; PQ is (def. 1) the *tangent-chord* of D, GH that of L; and (def. 2) V is the *vertex* to the *base* LD. If a diameter FB produced cut LD perpendicularly in D, and if, in DF, be taken DM equal to DP, therefore* a mean proportional between DF and DB, M is (def. 3) the *mean point*.†

Prop. I. Fig. 1, 2. In a given circle, one vertex has only

* III. Eucl. prop. 36, and VI. 14. Simson's edition, 1762, is referred to throughout.

† Other Definitions after Prop. VII.

one base; a right line indefinite in length: and one base has one vertex.

Thus: if GH and any other chords pass through the vertex V; the tangents from their extremities meet respectively in points of one indefinite line LD: and, conversely, if tangents be drawn from L and other points of LD; all their tangent-chords pass through one point V.

Demonstration. Fig. 2. Let E be the centre, and PQ the chord bisected in V. Draw the tangents GL, HL, PD, QD; and draw DE, DL, LE, EP, EQ, EH. On the diameters DE, LE, describe circles. That on DE passes through P and Q, because of the right angles DPE, DQE; and that on LE through G and H. And DE passes through V, because of the similar and equal right-angled triangles DEP, DEQ. By the circle on DE, the rectangle DVE is equal to PVQ; which, by the given circle GPQ, is equal to GVH; whence GEHD is* a circle, and is that on LE, and LDE is† a right angle. In like manner, if any other chord ST pass through V, and if tangents from S and T would meet in R, RDE would be proved a right angle. Therefore LDR is one right line. And conversely (fig. 1), if L be any point in a base to V; its tangent-chord GH passes through V. If not, let another chord G*h* so pass. Then (as proved) the tangent from *h* meets GL in that base, therefore in L; that is, in the same point as the tangent from H. Which is impossible. Therefore GH passes through V. *Q. E. D.*

Cor. 1. Fig. 1. The indefinite base to a given vertex V, is a line parallel to the chord bisected in V, and passing through any point, L or D, in whose tangent-chord V is.

* By a necessary converse of III Eucl. prop. 35. † III Eucl. prop. 31.

And the vertex to a given base LR, if E be the centre and ED a perpendicular on LR, is the intersection of ED with the tangent-chord of any point D or L of the base.

Cor. 2. Fig. 1. The mean point M, to a given base, is in the same diameter with V: and DM, as equal to DP, is a mean proportional between DE and DV. For, if EP be drawn, the triangles EDP, PDV, are similar.

Cor. 3. Fig. 2. If a chord GH bisect PQ the tangent-chord of any point D; DE bisects the angle GDH. For, draw DG, DH, EG. The right lines EG, EH, are equal; and the arcs EG, EH; whence the angle GDE is* equal to EDH. And, conversely, if DE bisect GDH, GH bisects PQ the tangent-chord of D. For, if any other chord through G should bisect PQ, the same angle GDE would be equal to one greater or less than EDH: which is impossible.

Cor. 4. Fig. 2. V is the vertex to the base LR in every circle whose centre, as E, is in DV produced beyond V, and whose radius, as EP or EQ, is found by the intersection P or Q of a circle on the diameter ED with the parallel PQ. For DPE and DQE are† right angles; whence DP and DQ are‡ tangents.

Prop. II. Fig. 3. The distance of an assumed point, in a given base, from the mean point within the circle, is equal to a tangent from the assumed point.§

Thus: if L be assumed in the base LD, and if M be the mean point within the circle GBQ, and LG a tangent; LM is equal to LG.

* III Eucl. prop. 27. † III. Eucl. prop. 31. ‡ III. Eucl. prop. 16. Cor.

§ This is a lemma in Hamilton's Stereography, with a different proof. B. III. Sect. I. L. 18.

Dem. From the centre E draw ED perpendicular to LD; and draw the tangent DQ; also EQ, EG, LE. The squares of LM and radius are equal to those of LD and DM (or DQ) and radius; therefore to those of LD and DE, so to that of LE or those of LG and radius; whence the square of LM is equal to that of LG, and LM to LG. *Q. E. D.*

Prop. III. Fig. 4. If the tangent-chord of any point will, produced, pass through any other given point; that of the second will pass through the first; and the squares of the two tangents, one from each point, are together equal to that of the intercepted right line; and, such line being taken as a base, and the mean point to it being found, lines to this from the two points contain a right angle.

Thus: if ST, the tangent-chord of R, will pass through L; GH, that of L, will pass through R; and the squares of the tangents LG and RS are together equal to that of LR; and, LR being a base, LMR at the mean point is a right angle.

Dem. Draw LH; and let it be between LR and LG. Draw from the centre E, to LR, a perpendicular ED; also GD, GR. Describe the circles LGEH, RSET *. These will pass through D, because of the right angles LDE, RDE; whence the rectangle RLD is equal to SLT, so to the square of LG, and LD is to LG as LG to LR. Therefore the angle LGR is † equal to LDG, to LHG, to LGH; and GH produced will pass through R.

And, because of the circle GHD, the rectangle LRD is equal to GRH, so to the square of RS; whence this square and that of LG (proved equal to RLD) are ‡ together equal to that of LR.

* As LGEH in Prop. I. † VI. Eucl. prop. 6. ‡ II. Eucl, prop. 2.

And LM is * equal to LG, and RM to RS; whence the squares of LM and RM are together equal to that of LR. Therefore † LMR is a right angle. *Q. E. D.*

Cor. 1. If V be the vertex to LR, and right lines from L and R to E, also to V, be drawn or conceived; the angle LER is acute, LVR obtuse. ‡

Cor. 2. If the tangent-chord of a point L will pass through a point R, and if D in LR make RLD equal to the square of LG; ED is perpendicular to LR. For, draw EH. Then RL is to LG as LG to LD; whence the angle LDG is § equal to LGR or LGH, so to LHG, and LGEHD is one circle. But LHE is a right angle; therefore also LDE.

Prop. IV. Fig. 4. If two points be such, that, the intercepted right line being a base and the mean point to it being found, lines from this to the given points contain a right angle, or that the squares of the tangents from the two points are together equal to that of the intercepted line; the tangent-chord of each of the two points will pass through the other.

Thus: if, LR being a base, LMR at the mean point be a right angle, or the squares of LG and RS be together equal to that of LR; the tangent-chord of L will pass through R, and that of R through L.

Dem. On the first supposition, the tangent-chord of R will pass through L: else it would cut the indefinite base in some other point, from which a right line to M would be || perpendicular to MR; which is impossible. Therefore also ¶ the tangent-chord of L will pass through R.

On the second supposition, LMR will be ** a right angle;

* Prop. II. † I Eucl. prop. 48. ‡ I Eucl. prop. 21.
§ VI Eucl. prop. 6. || Prop. III. ¶ Ib. ** *Dem.* of Prop. III.

whence (by the present proposition) the tangent-chord of each of the two points will pass through the other. *Q. E. D.*

Cor. If either LMR be a right angle, or the squares of LG and RS be together equal to that of LR; the other of these two follows from it.*

Prop. V. Fig. 5. If a diameter be cut by a perpendicular from any point of the circumference, and, produced, be cut by a tangent from the same point, and if an angle be made at any point of the circumference, by right lines to the two intersections; this angle and its supplement are bisected by right lines to the extremities of the diameter.

Thus: if LR, a diameter of GNH, be cut in I by NI, and, produced, in K by NK, and if GK cut the circumference in H between G and K; the angles INK, IHK, IGK, are bisected by NR, HR, GR, and their supplements INX, IHY, IGY, by NL, HL, GL; KN being produced to X, and KG to Y.

Dem. Produce NI to O in the circumference, and draw OR, OL. Then RNK is † equal to NOR, so to ONR or INR; also LNX to NOL, so to ONL or INL. Produce GI, HI, to T, S, in the circumference. Draw HT, SK, SR, SG, SL. Then, since HS and GT bisect NO, HKR or GKR is ‡ equal to RKS or RKT, HK § to TK, GK to SK; and HT and GS are perpendicular to LR. IHR or SHR is equal to SGR, so to GSR, which, having the supplement ‖ RHG, is equal to RHK. And, from the equal arcs RH, RT, TGR or IGR is equal to RGH or RGK. And the right lines LG, LS, are equal, and the arcs LG, LS; whence LHS or LHI is equal to LHG or LHY.

* Prop. III. and IV. † III Euct. prop. 32. ‡ Cor. 3 to prop. I.
§ III Euct. prop. 3. ‖ III Euct. prop. 22.

And LGI (LGS and IGS) is equal to LSG and ISG together, that is, to LSI, which, having the supplement LGH, is equal to LGY. *Q. E. D.*

Cor. 1. The angle HIT is bisected by LR, which bisects HT perpendicularly: and GIH is bisected by NO, because HIR is equal to TIR, so to GIL.

Cor. 2. In every triangle in which IK is one side, and the opposite angle is at the circumference of the circle, the two other sides, adjacent to IR and RK respectively, are * in the ratio of IR to RK.

Cor. 3. Fig. 6. If a right line IK be divided unequally in R, in a ratio given; a circle LNR may be found such, that, IK being a side of any triangle having the opposite angle at the circumference, one of the other sides shall bear to the remaining side the given ratio. Describe the semicircle ITK, draw RT perpendicular to IK, and the tangent TL cutting KI produced in L: a circle LNR, on the diameter LR, is the circle required. For, let the two circumferences cut each other in Q; bisect IK in F, LR in C; draw IN perpendicular to LR; also FQ, CQ, FT, CN, KN. Then, since LTF and TRF are right angles, the rectangle LFR is † equal to the square of FT, so of FQ; whence FQ touches ‡ LNR, FQC is a right angle, and CQ touches § ITK; whence the rectangle KCI is || equal to the square of CQ, so of CN, and the angle KNC, as equal ¶ to NIC, is a right angle, therefore KN a tangent.

Prop. VI. Fig. 5. If the tangent-chord of a given point be

* VI Euc. prop. 3. † VI Euc. prop. 8. Cor. and prop. 17.
‡ III Euc. prop. 37. § III Euc. prop. 16. Cor. || III Euc. prop. 36.
¶ VI Euc. prop. 14 and 6.

cut by a chord produced to that point; the produced chord is divided in harmonical proportion.*

Thus: if NO the tangent-chord of K be cut in V by a chord GH, or in I by the diameter LR, each, when produced, passing through K; then GK is to HK as GV to VH, and LK to RK as LI to IR. That is; the distances of G and H, of L and R, from K, are proportional to their distances from V and I respectively.

Dem. Let the figure be as in Prop. V.; and let GA, HB, be perpendiculars on LR. Then the angles HIR, GIL, are† equal. Therefore the triangles HIB, GIA, are similar; and GK is to HK as GA to HB, as AI to IB, as GV to VH. Further. LK is to RK as the rectangle LKR or the square of NK to the square of RK, as that of‡ NI to that of IR, as§ LI to IR. *Q. E. D.*‖

Cor. 1. If a chord GT, through any vertex I, will, produced, cut KC, the indefinite base to I, in any point C; then GC is to TC as GI to IT, and GC to GI as TC to TI. For the tangent-chord of C passes¶ through I.

Cor. 2. If GT be a chord through any vertex I, and Z any point in the base, and if the chord bisected in I be cut in D by ZG, in W by ZT produced; ID is equal to IW. For DW

* This proportion has been expressed in different ways. A line LK consists of three parts, LI, IR, RK; and the whole line is to either extreme part, as the other is to the middle part. Otherwise: LK, LR, LI, are three lines; and the first is to the third, as the difference of the first and second is to that of the second and third.

† In Cor. 1 to prop. V.

‡ Cor. 2 to prop. V.

§ VI EUCL. prop. 8. Cor. and prop. 20. Cor. 2.

‖ Among the proofs which might be given, of this proposition, one (applicable both to GH and LR) is by VI EUCL. prop. 3 and prop. A following it, together with the Prop. V here given and its first corollary.

¶ Prop. I.

is parallel to KC: and CZ is to ID as GC to GI, as * TC to TI, as CZ to IW. And, conversely, if ID be equal to IW, then GD and WT, produced, will meet in a point Z of the base.

Cor. 3. Fig. 6. If L, I, R, K, in that order, be points in one right line, and LK be to RK as LI to IR, and if LR be a diameter of a circle LNR, and IN perpendicular to LR; KN is a tangent. For, if not, the tangent-chord of K will cut LR in some point X other than I. Then LX is to XR as LK to RK, as LI to IR: which is impossible. So, if IK be the diameter of ITK, and RT a perpendicular; LT is a tangent.

Cor. 4. Fig. 6. If any three of L, I, R, K, be given; the fourth, if an extreme point, as K, may be found by LNR, IN, NK; if an intermediate point, as R, then by ITK, LT, TR.

Cor. 5. Fig. 7. A circle PHF may be described, having a given vertex V to a given base LR, and passing through a given point P whose perpendicular distance from LR is greater than from VF parallel to LR. Let GD be a perpendicular, through V, on LR. Draw PD, PV; also PG, making the angle VPG equal to PDG. G is the centre required. For, draw GF. Then, by the similar triangles DGP and PGV, the rectangle DGV is equal to the square of PG or GF; whence the triangles DGF and FGV are similar, DFG is a right angle, as equal to FVG, and DF a tangent; whence V is † vertex in PHF to LR. Let DV cut the circle first in H, then in K. DK is to KV as DH to HV. Therefore DH is greater than HV; and, if P were not further from LR than from VF, the thing could not be done. If P be given in the

* Cor. 1. † Cor. 1 to prop. I.

perpendicular, as at H or K, the circle is found from the proportion, DK to KV as DH to HV.

Cor. 6. Fig. 7. If two circles, SQT and HFK, have the same vertex V to a given base LR; they are not concentric, and one includes the other and does not touch it. Thus: if E be the centre of SQT, and G of HFK, and EV be greater than GV; SQT includes HFK, and does not touch it. For, let V D be the perpendicular on LR. Draw VQ parallel to LR, cutting SQT in Q and HFK in F. Draw DQ, DF: which are tangents. Then, if any point X were the common centre of the circles, and XF and XQ were drawn, both XFD and XQD would be right angles: which is impossible. E and G are * in DV produced: which line will cut SQT first in a point S, then in a point T; and in like manner cut HFK, in H and K. Then DK is to KV as DH to HV, and DT to TV as DS to SV. But DK is less than DT; whence the ratio of DK to KV is greater than of DT to TV. Therefore that of DH to HV is greater than of DS to SV; whence SV is greater than HV. *A fortiori* the whole of SQT falls without HFK.

Prop. VII. Fig. 7. *Problem.* Two circles, not concentric, being given, of which one includes the other and does not touch it; to find a base, having a vertex common to the two circles.

Solution. Let E be the centre of the outer circle SQT, and G that of the inner HFK. Draw EG indefinite; cutting the circles in H and S beyond G, in K and T beyond E. Find a right line A equal † to $\frac{\text{sq. ES} - \text{sq. GH}}{\text{EG}} - \text{EG}$; and divide it ‡

* Cor. 1 to prop. I. † See I EUCL. prop. 47; and VI EUCL. prop. 17 and 11.
‡ Cut it perpendicularly by a chord equal to 2GH, in a circle on the diameter A.

into two parts whose rectangle shall be equal to the square of GH. Make GV equal to the less part, and GD to the greater; taking V and D in EG produced beyond G. Through D, perpendicular to GD, draw LR the base required: to which V is the common vertex.

Dem. First, A is greater than 2GH. For sq. ES — sq. GH is greater than sq. EH — sq. GH, which is * sq. EG + 2EGH; whence A is greater than $\frac{\text{sq. EG} + 2\text{EGH}}{\text{EG}}$ — EG, which is 2GH. Therefore A can be divided as required.

Through V draw VQ parallel to LR; cutting HFK in F, and SQT in Q. Draw GF, FD, EQ, QD. Then, because the rectangle DGV is equal to the square of GH or GF, the triangles DGF and FGV are similar, the right angle GVF is equal to GFD, DF is a tangent, and V the vertex † in HFK to the base LR.

Further. To A (DG + GV) add EG. Then $\frac{\text{sq. ES} - \text{sq. GH}}{\text{EG}}$ is equal to DE + GV; whence sq. ES = DEG + EGV + sq. GH (DGV), = DGE ‡ + sq. EG + EGV + DGV, = $\overline{\text{DG} + \text{EG}} \times \overline{\text{EG} + \text{GV}}$, = DEV. Since, then, the square of ES or EQ is equal to DEV, DQ (as DF before) is a tangent, and V the vertex in SQT to LR. *Q. E. D.*

EV and ED might have been found, instead of GV and GD; *mutatis mutandis:* by making A equal to $\frac{\text{sq. ES} - \text{sq. GH}}{\text{EG}}$ + EG, and the rectangle under its parts equal to the square of ES.

* II Euct. prop. 4. † Cor. 1 to prop. I. ‡ II Euct. prop. 3.

Additional Definitions for the Introductory Propositions.

Def. 4. Fig. 8. The *points of concourse*, of a trapezium, are the two points in which (if at all) the opposite sides, produced, meet respectively. Thus: if ABCF be a trapezium, and if FA and CB meet in L, AB and FC in R; the points of concourse are L and R.

Def. 5. The *connecting line* is a right line joining the two points of concourse; as LR. And, if the figure be one inscribed in a circle, LR is cut in the *dividing point* D by a perpendicular from the center.

Def. 6. The *near* angle and the *remote* angle, of a trapezium having two points of concourse, are those, respectively, whose angular points have the least, and the greatest, perpendicular distances from the connecting line. The two others are the *mean* angles.

Prop. VIII. Fig. 8. The near angle, of an inscribed trapezium having two points of concourse, is greater than any of the others; and the remote angle is less than any.

Thus: if the figure be as in def. 4, and ABC be the near angle, AFC the remote; ABC is greater than each of BAF, BCF, AFC; which last is less than each of BAF, BCF.

Dem. BAF, having * the supplement BCF, is equal to RCB; and BCF to LAB. Therefore ABC, an exterior angle of the triangles RCB and LAB, is greater than either of the mean angles at A and C: and their equals, RCB and LAB, being exterior angles of the triangles LFC and RFA, are severally greater than AFC. *Q. E. D.*

* III EUCL. prop. 22.

Cor. 1. ABC is always obtuse, and AFC acute; because their sum is two right angles: whence AC is not a diameter.

Cor. 2. BF, subtending the mean angles, is greater than the other diagonal. For, if they be right angles, BF is a diameter. If they be not, one is acute; as BAF. Then, since this is greater than AFC, the double of it, that is, an angle on BF at the * centre, is greater than one on AC at the centre; therefore † BF than AC.

Cor. 3. The greater diagonal, produced, cuts LR between L and R; the less beyond L or R, if at all.

Cor. 4. The sides containing the remote angle, are greater than their opposites; FC than AB, AF than BC. For the triangle LAC is similar to LBF, and LCF to LAB. Therefore LF is to LC as BF to AC; whence LF is ‡ greater than LC, and than LB. But LF is to LB as FC to AB. By the like proof, AF is greater than BC.

Prop. IX. Fig. 9. The tangent-chord of each point of concourse, of an inscribed trapezium, will, produced, pass through the other.

Thus: if L and R be the points of concourse; the tangent-chord of L will pass through R, and that of R through L.

Dem. Let ABCF be the trapezium, ABC the near angle, AFC the remote, and L the concourse of FA and CB. Describe the circle ABL, meeting LR in D; and draw AD, also the tangents LG, RS. Then the angle LDA is equal to LBA, which, by the common supplement ABC, is equal to LFC or LFR; whence the triangle LDA is similar to LFR, and the rectangle RLD equal to FLA, so to the square of LG. But LRD is equal to ARB, so to the square of RS. Therefore

* III Eucl. prop. 20. † I Eucl. prop. 24. ‡ Cor. 2.

the squares of LG and RS are together equal to RLD and LRD, so to the* square of LR; whence the tangent-chord of L will† pass through R, and that of R through L. *Q. E. D.*

Cor. 1. If E be the centre, V the vertex to LR taken as a base, and M the mean point, and if right lines be drawn or conceived, from L and R, to E, to V, to M; LMR is a right angle, LVR obtuse, LER acute.‡

Cor. 2. D is the dividing point.§

Cor. 3. Each side of ABCF is a chord in a circle passing through the opposite point of concourse and the dividing point. For, describe the circle FCL; and let D be now the point in which this circumference cuts LR; and draw DF. Then the angle LDF is equal to LCF, so to LAR; whence the triangle LDF is similar to LAR, and the rectangle RLD equal to FLA, and so to the square of LG, as before. Therefore D, as‖ before, is the dividing point. And a circle CBR would be proved as ABL, and FAR as FCL, to pass through that point.

Cor. 4. The diameter LI of ABL is perpendicular to FC. For, let them meet in K; and draw ID. Then the angle LID is equal to LAD, so to LRF or LRK; whence LKR, as equal to LDI, is a right angle. In like manner the diameter of FCL would be proved perpendicular to AB, that of CBR to AF, that of FAR to BC.

Prop. X. Fig. 9. If an inscribed trapezium have two points of concourse; the connecting line is the base to which the mutual intersection of the diagonals is vertex in the inscribing circle.

Thus: if ABCF have L and R its points of concourse; LR

* II Euct. prop. 2. † Prop. IV. ‡ Prop. III, and its Cor. 1.
§ Def. 5, and Cor. 2 to prop. III. ‖ This Prop. and Cor. 2.

is the base to which V, the intersection of AC with BF, is vertex in the circle ABCF.

Dem. Let ABC be the near angle, AFC the remote, E the centre, D the dividing point, and A in LF. Draw ED, DA, DC, DB, DF. Draw or conceive the circles * ABDL &c. Then, by ABDL, the triangle LDA is † similar to LFR; and, by CBDR, RDC to RFL: whence the angles LDA, RDC, are equal, and ADC is bisected by DE. Further. If BF be a diameter, it coincides with ED; which ‡ bisects the tangent-chord of D. Let it be not a diameter. By the circle ABDL, the angle LDB has the supplement LAB, and is equal to BAF; and, by FADR, RDF is equal to RAF or BAF: whence LDB, RDF, are equal; also LDF, RDB; and BDF is bisected by DE. Therefore the tangent-chord of D is § bisected by AC and BF, in V. But ED bisects it: whence V is its intersection with ED, therefore || the vertex to LR. *Q. E. D.*

Cor. 1. Fig. 8. If two sides, as GS and HT, be parallel; a line KC parallel to them is the base. For, let GH and ST meet in K, GT and SH in I. Then one diameter LR bisects GS and HT perpendicularly, and, produced, bisects the angle GKT or HKS; whence GT and SH bisect ¶ the tangent-chord of K in I; which is perpendicular to LK or LR, and so is parallel to GS and HT; whence KC is ** base to the vertex I. Also GH and ST are equal.

Cor. 2. Fig. 9. DAEC, DBEF, are circles; except BF be a diameter of the inscribing circle. For the rectangle DVE

* Cor. 3 to prop. IX.
† *Dem.* of prop. IX.
‡ *Dem.* of prop. I.
§ Cor. 3 to prop. I.
|| Cor. 1 to prop. I.
¶ Cor. 3 to prop. I.
** Cor. 1 to prop. I.

is equal to the square of* half of the tangent-chord of D; therefore to AVC, and to BVF.

Cor. 3. Fig. 10. If ABCF be an inscribed trapezium, having V the mutual intersection of the diagonals, L and R the points of concourse, ABC the near angle, AFC the remote, and A in LF, and if FB and AC, produced, cut LR in I and K; LK is to RK as LI to IR. For, let LB and RB, produced, cut SP, drawn through V parallel to LR, in P and S. Then LK is to PV as CK to CV, as† AK to AV, as RK to SV; whence LK is to RK as PV to SV, as LI to IR.

Cor. 4. Fig. 10. If to M, the mean point to the base LR, be drawn LM, IM, RM, and KM produced to any point X; the angle IMK is bisected by MR, and IMX by ML. For, on the diameter LR, and on the same side with the trapezium, describe the semicircle LNR, and draw IN perpendicular to LR. Then KN, being drawn, is‡ a tangent. And, because LMR is§ a right angle, M is in the circumference LNR; whence|| the angles are bisected.

Cor. 5. Fig. 10. Or, if a base LK be cut in any two points I and K by chords, as FB and AC, through its vertex V, and M be the mean point; the angle IMK is bisected by drawing either AB or FC, producing it to R in the base, and drawing MR. For AB and FC meet the base in one point: and the inscribed trapezium ABCF may be completed. Also IMX might be bisected in a similar manner.

Cor. 6. Fig. 9. If any polygon, having an even number of sides, be inscribed in a circle, and its diagonals joining opposite¶

* Of PV in Fig. 2. † Cor. 1 to prop. VI. ‡ Cor. 3 to prop. VI.
§ Cor. 1 to prop. IX. || Prop. V.
¶ By *opposite* angles and sides are meant such, that the two intercepted arcs contain equal numbers of angular points.

angles meet in one point V, not the centre; the opposite sides will respectively meet, if at all, in points of the base to the vertex V. For any two opposite sides, as AB and FC, may be made the opposite sides of an inscribed trapezium having the same intersection of its diagonals. If two opposite sides be parallel, the base is * parallel to them.

Cor. 7. Fig. 11. If ABCF and LR be as in the proposition, and if IK, KN, NO, OI, be tangents respectively at A, B, C, F; the diagonals IN, KO, of the circumscribed figure IKNO, cut each other in the same point V as do those of ABCF, and are in the same indefinite lines as the tangent-chords of L and R. For, since AB the tangent-chord of K will pass through R, that of R will † pass through K; likewise through O, whose tangent-chord is FC. And it will pass through ‡ V, and § L. In like manner the tangent-chord of L will pass through N, I, V, and R.

Cor. 8. Fig. 11. Therefore the opposite sides of a circumscribed trapezium, if they meet, meet in points of the same indefinite base as that of such inscribed one; namely, whose angular points are the points of contact of the circumscribed. For the sides are tangents from chords through V. If two opposite sides be parallel, the base is parallel to them. For, if IO and KN, which are tangents at F and B, were parallel, FB would be a diameter, and perpendicular to them: and, because it passes through V, it would be the diameter perpendicular to LR.

Cor. 9. Fig. 11. Also, if a circumscribed polygon have an even number of sides, and its diagonals || pass through one

* Cor. 1. † Prop. III. ‡ Prop. I.
§ Prop. IX. || As in Cor. 6.

point V, not the centre; the opposite sides, if they meet, meet respectively in * one base; and it is the base in which those of such † inscribed one meet. If two opposite sides be parallel, the base is parallel to them.

Prop. XI. Fig. 12. If, in each of two circles having a common vertex to a given base, a chord be drawn through the vertex, and if, by joining the extremities of the two chords, a trapezium be formed having two points of concourse; the base is the connecting line. If two sides of the trapezium be parallel, they are also parallel to the base.

Thus: if, in BFT and ACK, BF and AC be chords through V the common vertex to the base LR, and if ABCF have two points of concourse; these are in LR. If FA and CB were parallel, they would be parallel to LR.

Dem. Through V draw NO parallel to LR. The opposite sides FA and CB, since they meet, cannot both be parallel to LR. Let FA, produced beyond A, meet LR in L, and cut NO in N. Through L and B draw LO, cutting NO in O; and draw LC, OC. Then, by the circle BFT, NV and VO are ‡ equal. And, because they are equal, then, by the circle ACK, AN (or FA) and OC produced will meet § in the base, therefore in L; whence OC and CL are one right line OL; of which, LB is part. Therefore LBC is one right line, and L a point of concourse. In like manner, some point R of LR is the other point of concourse. Therefore LR is the connecting line. And since, if FA or CB meet LR, they are thus proved to meet each other, therefore, if they do not meet each other, neither of them meets LR; that is, they are parallel to it. *Q. E. D.*

* Prop. I. † See Cor. 6. ‡ Cor. 2 to prop. VI. § Ib; converse.

Cor. 1. If the two circles coincide, ABCF becomes an inscribed trapezium: and thus is obtained a separate proof of the tenth proposition. *

Cor. 2. If, in any number† of circles having a common vertex and base, chords through the vertex be made diagonals of any trapeziums, inscribed or not; such base is the locus of all the points of concourse.

Prop. XII. Fig. 12. If a non-inscribed ‡ trapezium have two points of concourse; the connecting line is the base to which the mutual intersection of the diagonals is vertex in two circles, in each of which one diagonal is a chord.

Thus: if ABCF have L and R its points of concourse; LR is the base to which V, the intersection of AC with BF, is vertex in the circles ACK and BFT.

Dem. Let VD be the perpendicular on LR. Let ABC be the near angle, AFC the remote, and A between L and F. Through V draw NO parallel to LR, and indefinite. And let AC be first supposed to cut NO, and C to be nearer to LR than A is. Let lines bisecting BF and AC perpendicularly cut DV in E and G. With the centres E and G describe the circles BFT and ACK. In both, V is vertex to the base LR. For two circles, through F and A respectively, can § be described, each having V its vertex to LR. If these do not pass through B and C respectively, let them cut FB and AC in other points, X and Y; and complete the trapezium AXYF.

* The former proof is retained, as independent of harmonical proportion (involved in this proposition), and of the deduction by the coincidence.

† See Cor. 4 to prop. I. 3 or Cor. 5 to prop. VI.

‡ That is, one which cannot stand inscribed in a circle.

§ Cor. 5 to prop. VI.

Then FX and AY are chords through V in two circles having V their common vertex to the base LR: whence YX produced will * meet FA in L. And FY and AX cut NO, therefore are not parallel to LR, therefore † not to each other, but meet ‡ in a point of LR. This they cannot do, unless X be at B and Y at C: for, since X and Y are such that YX is directed to L, FY and AX will, if not passing through C and B, either meet before they cut LR, or beyond it. Therefore no two circles, passing through F and A respectively, except BFT and ACK, have V their common vertex to LR. Therefore those two have.

Let now AC be conceived § to coincide with NO. Then a circle, having its centre any point in DV indefinite, might pass through A and C. If a centre be found, in DV, by drawing DA or DC, and, perpendicular to it, a line from A or C cutting DV, then the circle will have V its vertex to LR; because DA and DC will be tangents.

Therefore, whether AC cut NO or not, LR is the base to which V is vertex in the circles BFT, ACK. *Q. E. D.*

Cor. 1. If FA and CB were parallel; RL parallel to them would be the base to which V would be vertex, in two such circles. For, let VD be a perpendicular on LR; and E, G, X, Y, as before. Then, if YX could meet FA, it would be ‖ in RL. But RL does not meet FA. Therefore YX does not; but is parallel to FA and CB; whence AX and FY, which must ¶ meet in LR, must pass through B and C, and therefore meet in R; whence R is a point of the base to which V

* Prop. XI. † Ib; latter part. ‡ Prop. XI.
§ This may be conceived without an additional figure.
‖ Prop. XI. ¶ Ib.

is vertex both in BFT and ACK. And this base, being * perpendicular to the right line through E, G, and V, is RL.

Cor. 2. If ABCF be a trapezium; one of those circles, as BFT, unless it coincide with ACK, includes it, and without contact. †

Cor. 3. If ABCF be a trapezium, either inscribed or not, having two points of concourse, L and R; the part NO, of the parallel to LR through V, intercepted between two opposite sides, is ‡ bisected in V. So between AB and FC.

Cor. 4. Fig. 13; the same as fig. 12, so far as the letters are common. If ABCF be a trapezium, inscribed or not, having two points of concourse L and R, and if FB and AC be the diagonals and V their mutual intersection, VD a perpendicular on LR, and right lines be drawn from D to F, B, A, C; the angles BDF and ADC are § bisected by DV. For the chord bisected in V is the tangent-chord of D, in each of the two circles of the proposition.

Cor. 5. Fig. 13. If FB and AC, produced, cut LR in I and K; then FI is to BI as FV to VB, and AK to CK as AV to VC. ||

Cor. 6. Fig. 13. The connecting line of a trapezium, inscribed or not, is ¶ divided harmonically in the four points L, I, R, K, in which it is cut by the produced sides and diagonals. For the demonstration, of the corollary here referred to, may (by the present proposition and its fifth corollary) be applied to trapeziums not inscribed. **

Cor. 7. Fig. 6. Hence, if L and R be the points of concourse

* Cor. 1 to prop. I. † Cor. 6 to prop. VI. ‡ Cor. 2 to prop. VI.
§ Cor. 3 to prop. I. || Cor. 1 to prop. VI. ¶ Cor. 3 to prop. X.

** This sixth corollary is proved differently in HAMILTON's Stereography, B. III. Sect. I. Lemma 22. The remainder of that lemma is the present Cor. 5.

of any conceived trapezium, whose diagonals produced cut the base in I between L and R, and in K beyond R, and if, on the diameters LR and IK, be described the semicircles LNR and ITK, and if IN and RT be perpendicular to LR; KN and LT, being drawn, are tangents.*

Cor. 8. Fig. 12 or 13. If the diagonals of a non-inscribed polygon, having an even number of sides, meet in one point; the opposite sides will respectively meet, if at all, in the base to which that point is vertex in two or more circles. For the proof applied † to the inscribed polygon may, now, be extended to the non-inscribed. If two opposite sides be parallel, the base is ‡ parallel to them.

End of the Introductory Propositions.

On the Elliptic Representation of a Circle, upon a plane surface, by Perspective.

Definitions.

Although the reader is presumed to be acquainted with the principles of perspective; yet the definitions in that science have varied so much, that it may be right to prefix a few here. Some new terms are also introduced.

Def. 1. The *centre of the picture* is that point, in the indefinite plane of the picture, in which it is cut by a perpendicular from the eye.

Def. 2. The *directing plane* is a plane through the eye, parallel to the picture.

Def. 3. The *intersecting line* and *directing line*, of an objec-

* Cor. 3 to prop. VI. † Cor. 6 to prop. X. ‡ Cor. 1 to this prop.

tive plane, are those in which that plane is cut by the picture and directing plane respectively: and the *centres* of those two lines are the points in which they are cut by perpendiculars from the eye.

Def. 4. The *intersecting point* and *directing point*, of an objective line or of its representation, are those two points in which the objective line cuts the intersecting and directing lines respectively.

Def. 5. The *director* is a right line from the eye to the directing point.

Def. 6. The *central line*, in an objective plane, is a right line cutting the directing line perpendicularly in its centre.

Def. 7. The *centroïd*, of an objective circle, is the point represented by the centre of the representing ellipse.

Def. 8. *Conjugate chords* are chords represented by conjugate diameters of the ellipse.

For brevity, the *representation* of any point A or line AB in any figure, is called *a* or *ab*; although the *a* or *ab* should not appear in the figure: small letters every where denoting points in the picture, whether exhibited in the figures or not. Each figure, after the first, represents a single plane: which, except in fig. 4 and 7, is the objective plane.

Prop. I. Fig. 1. The perspective representation of an entire circle, is an ellipse.*

Dem. Let the circle ABC be an object, viewed by an eye placed at O. Let FD be the plane of the picture, MR the directing plane, DE the intersecting line of the objective plane, and LR its directing line. Conceive an indefinite line, passing

* In these propositions, *ellipse* includes the case in which the representation is a circle; except the contrary be expressed or implied: a circle being an ellipse with equal axes.

always through O, to be moved round, along the circumference ABC. A cone will thus be generated. And the section of this cone by FD is the perspective representation of the circle. But, since the circle is supposed to lie wholly beyond FD, from O, the plane MR, through O, parallel to FD, neither cuts nor touches the circle. Therefore * the section is an ellipse. *Q. E. D.*

Cor. If the eye remain fixed, and FD be moved, parallel to itself, till it cut ABC; then a *part* only of ABC is seen through FD: and its representation is part of an ellipse, and similar to the corresponding part of *abc*. For a given figure will have similar projections, formed by the same lines OA, OB, &c, on parallel planes.†

Prop. II. Fig. 2. The centroïd of a circle is the vertex to the directing line taken as a base.‡

Dem. Let GTH be the circle, LR its indefinite directing line, and V its vertex to the base LR. Then, whatever chord, as GH or ST, passes through V, its tangents meet § in a point of LR. Therefore they have the same director, and their representations are ‖ parallel; whence those of GH and ST are ¶ diameters of the representing ellipse, and V is the centroïd. *Q. E. D.*

Cor. 1. Hence, from the directing line, the centroïd is found.**

* By the known properties of the cone. See, in particular, prop. 81, 82, 83, of Conic Sections by the Rev. T. NEWTON, Fellow of Jesus' College, Cambridge; 1794: which is the treatise referred to in subsequent notes.

† If the directing line touch or cut the circle, the representation is, accordingly, a parabola or hyperbola. It is not the design of these propositions, to enter upon those cases. They may be seen elsewhere: particularly in HAMILTON's Stereography; where also are various particulars of the elliptic representation.

‡ See Def. 2, of introductory Propositions. § Introd. prop. I.

‖ BROOK TAYLOR's Perspective, 1719; Cor. 1 to Theo. V.

¶ By conic sections. A necessary converse of T. NEWTON's prop. 28.

** Cor. 1 to introd. prop. I.

Cor. 2. The centroid remains fixed, if the circle and directing line remain: although the eye move, in the same directing plane; or the picture move parallel to itself; or vary its angle with the circle, the eye so moving as to preserve the same directing line.

Prop. III. Fig. 2. In conjugate chords, each is the tangent-chord of the directing point of the other.

Dem. Let GH and ST be conjugate chords. Produce them to their directing points R and L. Then, because *st* * and *gh* are conjugate diameters, *st* is † parallel to the tangents to the ellipse at *g* and *h*: whence the tangents to the circle at G and H have ‡ the same director, and the same directing point L, with ST; or GH is the tangent-chord of L the directing point of ST. In like manner ST is the tangent-chord of R the directing point of GH. *Q. E. D.*

Cor. 1. If GH, the tangent-chord of any point L in the directing line, will cut that line in any point R; L and R will be the directing points of conjugate chords. For ST the tangent-chord of R will § pass through L; whence the tangents to the ellipse at *g* and *h* are parallel to *st*. And those at *s* and *t* are parallel to *gh*. Therefore *st* and *gh* are conjugate diameters.

Cor. 2. If M be the ‖ mean point, and L and R the directing points of conjugate chords, LMR is ¶ a right angle. And, if LR be the directing line and LMR a right angle, L and R will be the directing points of conjugate chords. For the tangent-chord of each point will pass ** through the other.

* See what is subjoined to the definitions. † By conic sections.
‡ A necessary converse of Cor. 1. to Theo. V of Brook Taylor.
§ Introd. prop. III. ‖ Introd. prop^n. def. 3. ¶ Ib. prop. III.
** Ib. prop. IV.

Cor. 3. If therefore one chord through the centroïd V were the diameter, consequently * perpendicular to LR and passing through M; the conjugate chord would be parallel to LR, therefore bisected in V.

Prop. IV. If any right-lined figure, having an even number of sides, be inscribed in a circle, or described about one, and if its diagonals, joining opposite angles, pass through the centroïd; the opposite sides of the representation are parallel respectively.

Dem. If the circle be parallel to the picture, the centroïd is the centre, and all the respectively opposite sides, of the inscribed or circumscribed figure, are parallel; therefore also their representations. Let the centroïd be another point. The opposite sides † will meet, if at all, in the indefinite base to the centroïd taken as a vertex, that is, in ‡ the directing line of the objective plane. If any two opposite sides will not meet, they are § parallel to the directing line. Therefore, in all cases, the representations are parallel. *Q. E. D.*

Cor. This is applicable, *mutatis mutandis*, to non-inscribed figures.‖

Prop. V. Fig. 3. If the centre of an objective circle be in the central line of its plane; one axis of the representing ellipse is parallel to the intersecting line, the other perpendicular.

Dem. Let V be the centroïd, A the centre of the directing line LR, and AO, in the central line, equal to the distance of A from the eye; BF the diameter through V, and PQ the chord bisected in V. Then, since FA cuts the intersecting line

* Cor. 1, 2, to introd. prop. I. † Introd. prop. X. and cor. 6, 8, 9.
‡ Prop. II. § Ib. with cor. 1, 6, 8, 9, of introd. prop. X.
‖ Introd. prop. XII, and cor. 1 and 8.

perpendicularly in its centre, therefore (by perspective) *fv* is perpendicular to that line, and *vq* parallel: whence *fvq* is a right angle. But no two conjugate diameters are at right angles, except the axes. Therefore *bf* and *pq*, always * conjugate diameters, are now the axes. *Q. E. D.*

Cor. 1. The perpendicular axis has (by perspective) its vanishing point in the centre of the vanishing line of the objective plane.

Cor. 2. It is also directed to the centre of the picture. For the directing line is † perpendicular to that plane, through the eye, which is perpendicular to the objective and directing planes. Therefore the central line is in that perpendicular plane: whence its indefinite representation is the common section of that plane with the picture; which common section passes through the centre of the picture.

Cor. 3. If O be at the mean point M (which is always ‡ in BF), in which case the distance of A from the eye is equal to a tangent from A; the representation is a circle, whose centre represents V. For, from the further extremity F of BF, draw FP; also FQ, produced to R in LR. Draw MR, OR, AQ. Then, if AOR, revolving on AR, bring O to the eye, AO and OR become the directors of *fv* and *fq*, therefore parallel to them respectively; whence the angle *vfq* is § equal to AOR, and the triangle *vfq* similar to AOR, wheresoever O is in AF. But, if O be at M, AO is equal to AR. For the angle AQR is equal || to FPQ, to FQP, to ARQ; whence AR is equal to

* Cor. 3 to prop. III.
† XI Euel. prop. 19.
‡ Cor. 2 to introd. prop. I.
§ XI Euel. prop. 10.
|| Its vertically opposite is so. III Euel. prop. 32.

AQ, so to AM; therefore *vq* to *vf*, and the axes are equal; but V remains the centroid.

Cor. 4. If AO and AM be unequal, the axes are so. If AO be greater than AM, the axis perpendicular to the intersecting line is the major axis; if less, the minor.

Cor. 5. Fig. 4. If O be at the mean point, the section of the cone of rays is the subcontrary * section. For, let ABF be as before, but O now at the real place of the eye; so that fig. 4 shall be the plane in which are BF and its representation *bf*. Draw AO, OF. Then *bf* is parallel to its director AO. And, since AO is equal to a tangent from A, its square is equal to the rectangle FAB; whence the triangle FAO is similar to OAB, and the angle OFA or OFB equal to BOA, so to O*bf*.

Cor. 6. Fig. 4. If OFB and O*bf* be unequal, the axes are so. If OFB be greater than O*bf*, the axis perpendicular to the intersecting line is the major axis; if less, the minor.† For, if A and B and F be fixed, whilst, by the motion of the eye directly from or towards A, the angle OFB and the length AO are varied; these will increase or decrease together, and O*bf* will accordingly decrease or increase.

Prop. VI. Prob. I. Fig. 3 and 5. A circular object being given in magnitude, and in position with respect to the eye and the plane of the picture; to find the axes of the representing ellipse.

Fig. 3. If the centre of the circle be in the central line of the plane, find (by perspective) *bf*, *pq*; which are ‡ the axes.

Fig. 5. If it be not so placed, let the circle GTH be the object, and E its centre. From the given position the

* Known in conic sections. See T. Newton, prop. 82.

† Cor. 4. ‡ Prop. V. and its *Dem.*

directing plane is determined, and the directing line. Let this line be LR, and A its centre. Make a right angle LAO, on the same side of LA as the circle is; and make AO equal to the distance of A from the eye. Find V the * vertex to the base LR; also M † the mean point. Draw OM; and bisect it perpendicularly in Q, by QC cutting LR in C: and draw CO. In LR take CL and CR, each equal to CO; and, through V, draw, towards R and L, the chords GH and ST. Find *gh* and *st*, by perspective. These are the axes.

Dem. With the centre C and radius CO, describe the semicircle LOMR; and draw LO, OR, LM, MR. Then, because V is ‡ the centroïd and LMR a right angle, *gh* and *st* are § conjugate diameters. And they are perpendicular to each other. For, if LOR, revolving on LR, bring O to the eye, OL and OR become the directors of *st* and *gh*, therefore parallel to them respectively; whence the angle *tvh* is || equal to the right angle LOR. But no two conjugate diameters are perpendicular to each other, except the axes. Therefore *gh* and *st* are the axes. *Q. E. D.*

In *practice*, find (by trial) in the directing line a point C equidistant from O and M; and, in the same line, take CL, CR, each equal to CO. And, if either L or R, suppose R, be at an inconvenient distance, the chord GH, directed to R, may be found as the tangent-chord of L. ¶

Cor. 1. Fig. 6; the same as Fig. 5, so far as the letters are common. One mode of finding the axes, after GH and ST found, is this. Draw GT, and produce it cutting LR in B. Draw OB; also BK, cutting OR perpendicularly in K. Find,

* Cor. 1 to introd. prop. I. † Cor. 2. ib. ‡ Prop. II.
§ Cor. 2 to prop. III. || XI Euct. prop. 10. ¶ Prop. III.

by perspective, *gt*, or *gv*, or *vt*; and, upon the line so found, make a triangle *gvt* similar to OKB, having the angle *g* equal to O, and *v* to K. Then are *gv* and *vt* two semi-axes. For R and B are the directing points of GV and GT.

Cor. 2. Fig. 5, 6. When E is not in the central line of the plane, neither GH nor ST is perpendicular to LR; therefore neither axis tends to the centre of the vanishing line of the plane.

Cor. 3. Fig. 5, 6. Wheresoever O is in the arc LMR, the same chords GH and ST are the originals of the axes.

Cor. 4. Fig. 6. The major axis has its directing point on that side, of the central line AO, on which E is placed. For, draw ML, MB, MR. Then, if O were at M, *gv* would be * equal to *vt*, and the angle *vtg* to *tgv*. Therefore LMB is † equal to BMR. But, if O be in the arc between L and M, the angle LOB stands on a greater arc in the circle LMR than LMB does, and BOR on a less than BMR; whence LOB, therefore OBK, is greater than BOK, OK than KB, and *gv* than *vt*.

Prop. VII. Fig. 7. If the object be a sphere in any oblique ‡ position; the major axis, of the representing ellipse, is directed to the centre of the picture.

Dem. To a given sphere conceive any tangents drawn from O the eye. The points of contact are in the circumference of a circle, whose representation (as to the outline) is that of the

* Cor. 3 to prop. V.

† Or, without perspective, these angles are equal; by Cor. 5 to introd. prop. X: where they are (fig. 10) IMR, RMK.

‡ By *oblique* position is here meant any except that in which a right line, from the eye to the centre of the sphere, passes through the centre of the picture.

sphere, and whose plane is perpendicular to a line * from O through E the centre of that circle to the centre of the sphere. If this line passed through the centre of the picture, such plane would be parallel to the picture, and the representation a circle. But, let it not so pass. Yet such plane is to be taken as the objective plane. And, since E is in every plane perpendicular to it through the eye, and the central † line is in one such plane, and both are in the objective plane, E is in the central line. Let FB, the diameter in that line, have B its extremity nearer to its directing point A, and be produced to A. Draw OF, OB, OA; and let the representation *fb* meet FB produced in K. Then the angle OFB, being equal to OBF, is ‡ greater than B*b*K or O*bf*. Therefore the axes are unequal, and the major axis is directed to the centre of the picture. § *Q. E. D.*

Cor. 1. If the plane of a picture parallel to the former should cut off part of FB; then the part viewed, of the circle or sphere, would have a representation ‖ similar to its former one, and the major axis would still be directed to the centre of the picture.

Cor. 2. The centre of the picture is not the vanishing point of the major axis. For FAO or FK*f* is not a right angle.

Lemma. Fig. 8. No. 1 and 2. If original lines, in one plane, converge to a point; their representations will converge to a point, or be parallel. And, if the representations converge to a point; the originals will so converge, or be parallel.

* The intersection of this line with the picture is also its vanishing point. When this is given, the vanishing line of the perpendicular plane is found by perspective. Brook Taylor, Prob. XV. † In Cor. 2 to prop. V.

‡ I Eucl. prop. 16. § Cor. 6 and 2 to prop. V. ‖ Cor. to prop. I.

Dem. Let the indefinite original lines CF, DG, EH, meet in P; having C, D, E, their directing points, and F, G, H, their intersecting points. From A the centre of the directing line, and towards FH, draw AO perpendicular to AD and equal to the distance of A from the eye. Draw CO, DO, EO: which are the directors, brought down, into the original or objective plane, by the revolution of COE or the directing plane on CE. Parallel to these draw lines through F, G, H: which will be the indefinite representations, brought down by the revolution of the picture on FH. These indefinite representations will form, on FG and GH, triangles respectively similar to COD and DOE; and perpendiculars from their vertices to FH will bear to OA the ratios of FG to CD and GH to DE. But these ratios are equal, because FH is parallel to CE. Therefore the perpendiculars are the same line KN, and the vertices are one point K. If P be in the directing line, by which C, D, and E, coincide with P; there is only one director, and the representations are * parallel. One part, then, is proved.

Now, let F, G, H, be the intersecting points, and FK, GK, HK, the indefinite representations, brought down as before: and let KN be a perpendicular on FH, and let AO be as before. Draw OC, OD, OE, parallel respectively to KF, KG, KH, and cutting the directing line in C, D, E; and draw, indefinitely, CF, DG, EH. These are the original lines represented; because C, D, E, are the directing points, and F, G, H, the intersecting points. From similar triangles, CD is to FG as CO to FK, as AO to NK, as EO to HK, as DE to GH; or CD is to FG as DE to GH. Therefore, if CD and FG be unequal, consequently DE and GH, then CF, DG, EH, will

* BROOK TAYLOR; Cor. 1 to Theo. V.

meet in a point P, of DG, making PD to PG as CD to FG. And, if CD and FG be equal, the three will either meet in a point equidistant from FH and CE, or be parallel. *Q. E. D.*

Cor. 1. If the originals neither converge to a point nor are parallel, the representations do not converge to a point.

Cor. 2. It is impossible, therefore, that the major or minor axes should so converge, of ellipses representing any circles; unless the originals of the axes should in any case so converge or be parallel: which will only be, if the intervals between their intersecting points be proportional to those between the directing points. *

Prop. VIII. Prob. II. Fig. 9. The magnitude and positions being given, of any equal circles in one plane, having their centres in the central line; to find a law which the axes, of the representing ellipses, observe in their directions.

One axis is † parallel to the intersecting line, the other perpendicular. Let AR, KI, AK, be the directing, intersecting, and central lines, indefinite; AK passing through the centres of the circles *i*, *ii*, *iii*, and any others such as supposed. Conceive O (not in the figure) at the real place of the eye. AO may be equal to a tangent from A to some one circle, or not. Let the *ii* be so placed. This alone will be ‡ represented by a circle: those § between KI and *ii* by ellipses whose minor axes are parallel to KI, and major axes directed to the centre of the picture; those beyond *ii* by ellipses whose major axes are parallel to KI, and minor axes directed to the centre of the picture. This, then, is a *law* observed.

* Yet some advantage may be gained, by discovering any *law* or *regularity* observed in the directions of the axes. This is attempted in the remaining propositions.

† Prop. V. ‡ Cor. 3 and 4 to prop. V. § Cor. 4 and 2 to prop V.

Cor. 1. The law holds in unequal circles, when all nearer and further than *ii* have, respectively, their mean points nearer and further than that of *ii*: because the distance of the mean point from A is equal to the tangent. But the inequality may be such as to destroy this supposition.

Cor. 2. If AO be less than AK, therefore less than the parallel and equal line (OS in Fig. 4) which is the distance of the vanishing line from the eye*, and if the mean point of the nearest circle be beyond the picture; then all the major axes are parallel to KI, all the minor directed to the centre of the picture.

Cor. 3. When it is known which axis is parallel to KI, then, if one point of either be given, its direction is got by drawing a line to the centre of the picture, or else a line parallel to KI.

Cor. 4. The extremities of the axes parallel to KI, are bounded by the following limits. Let PQ be the tangent-chord of A in the nearest circle, and GH the parallel diameter; G and P being on one side of AK. If lines be drawn through G and P, parallel to AK, their representations are the boundaries of the extremities on that side of AK; and the like on the other side. For PQ is the original of the axis: and the corresponding chord is greater, as the circle is more distant; but always less than the diameter.

Prop. IX. Prob. III. Fig. 10. To find the same, when the centres are in a line parallel to the central line.

Let one rank of equal circles so placed be *i*, *ii*, *iii*, &c.; let

* This supposition may, when the objective plane is perpendicular to the picture, be expressed thus: If the height of the eye be less than the distance of the picture. Where *heig bl* denotes perpendicular distance from any objective plane.

another be *ix*, *x*, *xi*, &c.: all having their centres E, E, &c. From the directing line AN, find the centroïds V, and the mean points M, in as many circles as may be requisite. On the same side of AN as the circles are, take AO, in the central line, equal to the distance of A from the eye; and make A the centre of a circle ON. In either rank, as *i*, *ii*, &c, if all the mean points be without ON, find * the directing point L of the minor axis of *i* the nearest circle. By the construction, the directing points of the minor axes for all the circles *ii*, *iii*, &c, will be between L and A. Draw LO. In the plane of the picture place the triangle LOA, or any similar triangle, in the same situation with regard to the indefinite intersecting line, or some parallel line, as LOA has here with regard to the directing line. Then † will all the minor axes, for *ii*, *iii*, &c, be respectively parallel to lines drawn from points in the base of such triangle to its vertex corresponding with O; each of such points will be nearer to the point corresponding with A, as the circle is further from the directing line; and the approach to A (and to its corresponding point) will be without limit, while circles are added. This, then, is a *law* observed by the minor axes: to which the major will be perpendicular, respectively.

If the circles be unequal, the law holds good; unless the inequality cause the mean point of a further circle to be not further than that of a nearer circle.

If the angle LOA be inconveniently large, find also the directing point F for the circle *ii*. Then all the directing points, for *iii*, &c, will be between F and A. Or, in some cases, find the directing point G for the furthest of the rank: and then the

* Prop. VI. † Ib.

directing points, for all between *i* and the furthest, will be between L and G.

The use of the law found, is this. When, in practice, it would be tedious to find * accurately, in position and length, the axes for each circle; the artist may, by this law, know pretty nearly the directions of the axes for all the circles, of the rank for which he has constructed the triangle LOA or FOA. Those axes, howsoever seldom they may converge † to a point, are always parallel to lines which do. In addition, he may find, by perspective, such particular points as any case may suggest. If ST be the diameter perpendicular to the directing line, *st* is readily found: which passing through *v*, so being a diameter of the ellipse, its middle point is one point in each axis. Or, some centroïds V may be found; whence the elliptic centres *v*, by easy operations of perspective. If the circles are equal, it may suffice, after V found in a few of the nearest circles, to take the remaining lines EV (in the same rank) equal, or to diminish them by the view. And, if the circles (in each rank) be also at equal intervals, their centroïds may often be considered as at equal intervals; or, after a few of the nearest. Whence the points *v*, with increased facility.

If the mean point in any circle, as *ix*, be within ON, find the directing point R of the major axis: which may be more convenient than that of the minor, as nearer ‡ to A. But this case cannot happen, with the mean point lying beyond a picture perpendicular to the objective plane; unless the perpendicular distance (by the scale assumed for any delineation) of

* By prop. VI. † See Cor. 2 to Lemma.

‡ If M be in the circumference ON, the directing points of the two axes are equidistant from A ⊥ by the construction of prop. VI.

that plane from the eye be greater than the distance of the picture.

If a figure, constructed as fig. 10, happen to be inconveniently large, it may be constructed upon a reduced scale. The triangles LOA &c. remain similar, each to itself; whence the directions of the axes are the same.

Prop. X. Prob. IV. Fig. 11. To find the same, when the centres are in a line parallel to the picture.

Of the circles *i*, *ii*, &c, let the centres be E, E, &c.; let AO and ST be as in Prop. IX, and D, D, the directing points of ST. In one circle, find the centroid V and the mean point M. Through these draw lines parallel to AD: which will cut the other lines ST in the points V and M of all. Let the parallel through M cut AO, or AO produced, in F.

First case. Let AO be less * than AF. Let *ii* be so placed, that OD, being drawn, may equal DM or AF. Find G the directing point of the minor axis for *ii*; and draw GO. Transfer † GOA to the picture; (or place therein a similar triangle.) Then the minor axes, for all the circles on the same side of AO as *ii* is, are parallel to lines from points of GA to O; and, the nearer any such circle is to *ii*, on either side of it, the nearer will the axis be to parallelism with GO. Nor is there any limit to the approach to parallelism with GO, or, reversely, with AO. This, then, is a law observed.

For the circle OM ‡ has the shortest radius when its centre is at the D of *ii*. That radius is increased, whether M (always

* This is always so, when the picture is perpendicular to the objective plane, if (by the scale) the perpendicular distance of the plane from the eye be less than the distance of the picture; unless a mean point be between the picture and AD.

† As LOA in prop. IX.

‡ Drawn as in prop. VI.

in the parallel through F) be taken between F and the M of *ii*, or beyond that M; and the circumference cuts GA between G and A, approaching to A as the radius is increased. And both the increase of the radius and the approach of the circumference to AO are without limit, while circles are added.

The *i* and *iv* are so placed, in the figure, as to have the same directing point for their minor axes; C being the centre of OMM.

Second case. Let AO be AF. Produce AG to H, making AH equal to AF. Then, the nearer M is to F, the less is the radius of the circle OM or FM; never less than AF. At the same time, its centre is the nearer to A, and the directing point of the minor axis is the nearer to H: which approaches may be made without limit. Place, as before, in the plane of the picture, a triangle; now HFA, or one similar. The minor axes will be parallel to lines from the base of such triangle to its vertex: each, as its circle is further, being nearer to a right angle with the intersecting line; and without limit. This is a law observed.

If requisite, find accurately the directing points of the minor axes for a few circles nearest to AF: or for one; as N for *iv*. In HFA, transferred to the picture, mark N. Then, for all the circles nearer than *iv*, the minor axes will be parallel to lines from points between H and N; and, for all beyond *iv*, between N and A.

In this second case, if M be at F, the representation is * a circle.

Third case. Let AO be greater than AF. With the centre A describe a circle LOR; cutting AD in R on the same side

* Cor. 3 to prop. V.

of AO with the objective circles before given, and in L on the other side. Then,* if one of those circles, as *ii*, have its mean point in the circumference OR, the directing points of the two axes are L and R. And, for any circle between *ii* and AO, the directing point of the major axis is between R and A, and nearer to A than that of the minor is; which is beyond L. And, for all circles beyond *ii*, the directing points of the minor axes are between L and A. Draw LO, RO. Then LOA, ROA, or triangles similar to them, transferred to the picture, as† GOA in the first case and HOA or HFA in the second, give lines to which the axes will be respectively parallel: and, of the minor axes, each, as its circle is further, is nearer to a right angle with the intersecting line; and without limit. This is a law observed.

Any requisite approximation to accuracy may be made; by the mode used in the second case, when N was found.

A figure, constructed as fig. 11, may, if required, be reduced by a scale.‡

Prop. XI, and last. Prob. V. Fig. 12, 13. To find the same, when the centres are in a right line oblique to the intersecting line of the plane.

Fig. 12. If the circles had been so placed that (instead of the centres) the mean points, M, M, &c, were in such line having any directing point X; a law observed would have been as follows. Let LR be the directing line; and let AO be as before, but less than AF, and in F let AO produced meet XM. With the centre A describe a circle, having the diameters ON, LR; L being on the same side of A with X: and let XM

* Construction of prop. VI.

† As LOA in prop. IX.

‡ See the end of prop. IX.

be wholly out of LOR. In FM take FP towards X, and FQ the contrary way, each a mean proportional between FN and FO; and draw PC and QK, perpendicular to PQ, cutting the directing line in C and K. Let the circle PG with the radius CP cut LR in G towards R, and let the circle QH with the radius KQ cut it in H towards L. Let the M nearest to X be between X and P. In the directing line find the centre W of a circle passing through O and this M, cutting FM in U also, and LR in I beyond A. Then, for circles whose mean points are U and the M nearest to X, the directing point of the minor axis is I. If a circle move, and so that its mean point shall move in XM; then, while this moves from that M to P, and on from P to U, the directing point of the minor axis moves accordingly from I to G, and back from G to I. While the mean point moves from U to F, from F to Q, then beyond Q without limit, such directing point, accordingly, moves from I to A, then from A to H, then approaches to A without limit. From these directing points triangles may be constructed*, and transferred to the plane of the picture; giving lines parallel, as before, to the minor axes. Such, then, would have been a law observed, if the mean points had been in a right line.

For, describe the circles PON, QON. Their centres are† C and K. Therefore they are the circles PG and QH; that is, the circles OM‡ with mean points at P and Q. Now, since all the circles OM have§ their centres in the directing line, and since, if a centre were assumed between C and K, the circumference through O could not meet XM, no centre is between

* As LOA, GOA, &c; in prop. IX, X.
† III Euel. prop. 1, 37, 19.
‡ According to the construction of prop. VI.
§ Ib.

C and K; and CP is the shortest radius, and, when M is beyond F, KQ is the shortest; and, the further any centre W is beyond C, the more does its circumference fall within the circle POGN at OIN, and the more without it elsewhere. And the like beyond K. Therefore, as the mean point moves from the first M to P, the circles OM become smaller, and cut LR in points further beyond A; which are the directing points of the minor axes; and the reverse takes place, while the mean point moves from P to U, and so on to F. As it approaches to F, the radius of OM increases without limit. When it is at F, the arc OM becomes a right line; which passes through A: whence the minor axis is * perpendicular to the intersecting line. While the mean point moves from F to Q, then beyond Q, the radius of OM decreases to KQ, then increases without limit; and the intersection of the circumference OM with LR moves from A to H, then approaches to A without limit.

The cases of AO equal to AF and greater, bear a resemblance to the second and third cases of Prop. X; and are not perhaps sufficiently frequent, in practice, to justify an examination here. And, whatever be the length of AO, if any mean point fall within LOR, it is to be remembered † that the directing point of the major axis is then nearer to A than is that of the minor.

Fig. 13. Now let the centres E, E, of the objective circles, be in a right line. Let LR and AO be as before; also the diameters ‡ ST perpendicular to the directing line; D, D, being their directing points, B that of the line through the

* As also by Cor. 4 to prop. V.

† Prop. IX, last paragraph but one; and X, 3d case. ‡ As in prop. IX, X.

centres, and H that of the line through the points T, the nearer extremities of ST. Let one of the circles have its centre in the central line; let F be its mean point, and AO less than AF. Let the circles *i*, *ii*, and any others, be on the same side of AO with B; and *ix* &c, on the other. Take any one, as circle *ii*, and draw TH; also HN, parallel to DT, cutting BE in N; and produce HB to Q, making BQ equal to HB. Then DT is to DH as DE to DB, as HN or TE to HB, as ES to BQ; whence DT is to DH as DS to DQ, and the rectangle SDT or the square of DM to QDH as the square of DT to that of DH, therefore as that of HN to that of HB. This is * a property of the hyperbola. Therefore all the points M are in the curve of an hyperbola, whose transverse axis is QH and conjugate axis equal to twice HN, that is, to a diameter ST; having the vertex H, and the † asymptote BE.

What follows is an attempt to make such approximation, to some law or regularity observed in the directions of the axes, as may be of use without constructing the hyperbolic curve.‡

As to the remoter circles, it may often be sufficiently accurate, in practice, to consider their mean points as in a right line; either drawn through one of them, parallel to BE, or drawn from F to the mean point of the furthest. So far as this can be admitted, the theory already laid down, respecting mean points in a right line, is to be taken also as the law or laws observed where the centres are so placed. If, in any case, one such line be not sufficiently accurate for all the circles

* T. Newton, prop. VI. † Ib. prop. XXII.

‡ The construction of prop. VI is to be kept in mind: referred to already in prop. IX, X, XI.

beyond F (from B), two such lines may be assumed, at different distances from BE; the one nearer to BE being for the remotest circles. As to mean points between F and H, yet not within the circle LOR with the centre A; find the minor axes, each separately, for as many, of the nearest to H, as requisite. If any remaining circles on that side of AF, as *ii* &c, should seem better referred to some law observed; then, with the centre W equidistant from O and the M of *ii*, describe a circle MOI cutting the directing line in I beyond A. If any mean point be within this circle, the directing point of the minor axis is beyond I. And, if any other objective circle have its mean point U in the circumference MOI, or if U in MOI can be found as the mean point of a circle purposely added or supposed (of the given magnitude, and having its centre in BE), then any mean points between that M and U will have the directing points of the minor axes beyond I. If how much beyond be inquired; the points P and G may be found, as (fig. 12) when the mean points were in a right line: and G will be a limit beyond which, at least, the directing points cannot go. And U will perhaps frequently be found with sufficient accuracy, if taken as the intersection of the right line MF with MOI. If the mean point come to U, the directing point returns to I; and if the mean point move from U to F, the directing point moves from I to A. Triangles constructed as * before, and transferred to the picture, will give lines which may be taken as parallel to the axes; though with some abatement from strict accuracy. But such is a law, or laws, which may be considered as observed, when the centres are in a right line.

* As LOA, GOA, &c; in prop. IX, X.

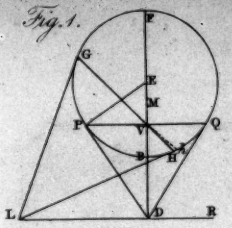
As to the cases of AO being AF and greater, and that of mean points within the circle LOR with the centre A; it seems unnecessary to add to what was said in concluding that part, of this proposition, in which the mean points were supposed to be in a right line.

Since the axes of the hyperbola are given in length and position, it could be described. And its intersections with the diameters ST would give all the mean points. Whether this would aid the perspective operations so much as to compensate for the labour of constructing the curve, I leave to others to examine.

RICHARD HEY.

March 1814. Hertingfordbury, near Hertford.

Fig. 1.



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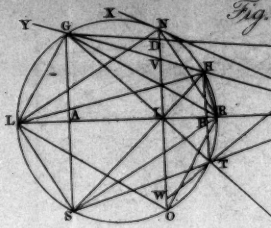

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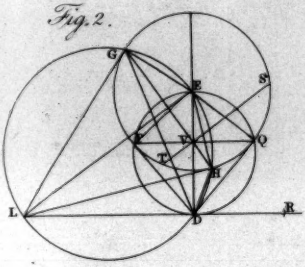

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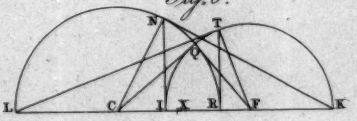

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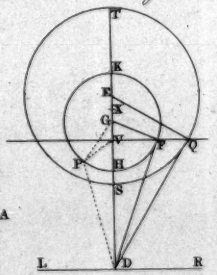

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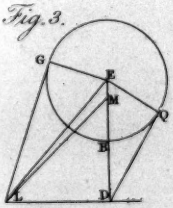

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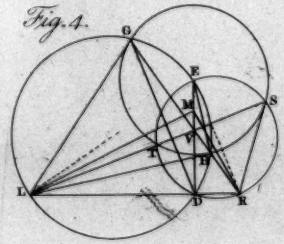

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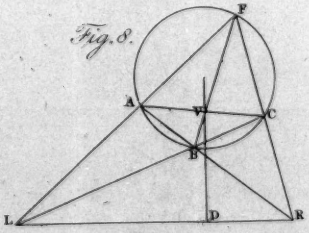

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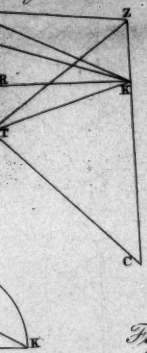
Fig. 5.



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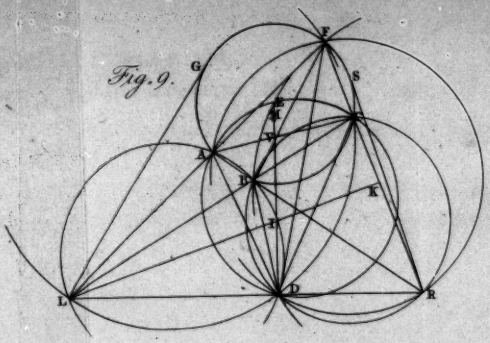

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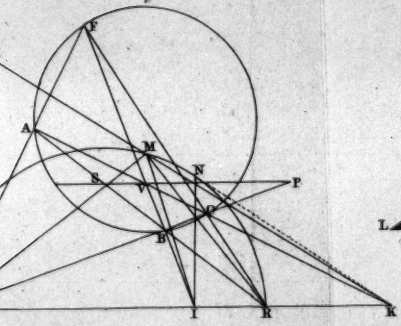

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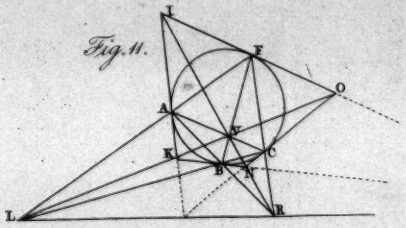

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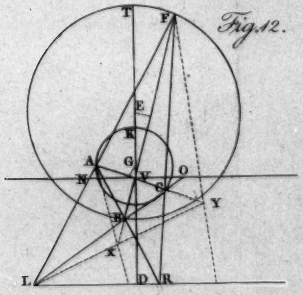

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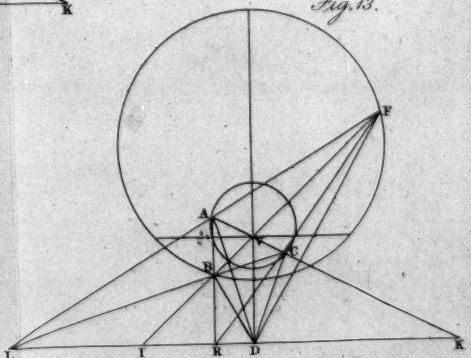


J.C. Bourne sc.

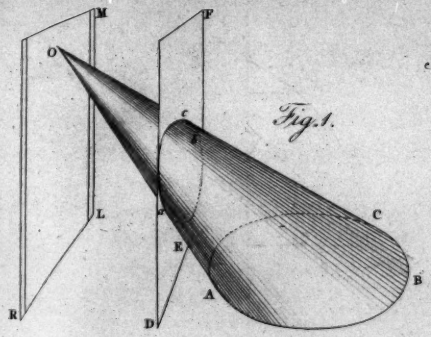
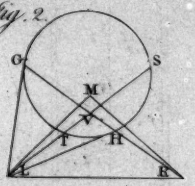


Fig. 1.



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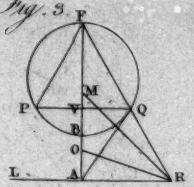

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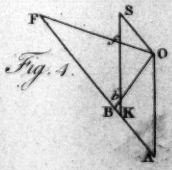

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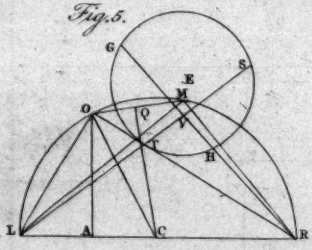

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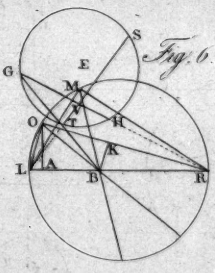

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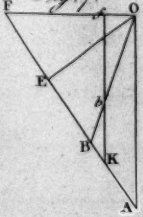

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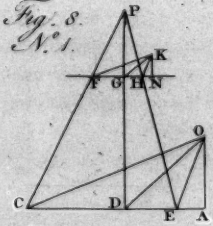

Fig. 8. N° 1.

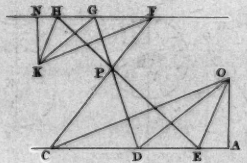

Fig. 8. N° 2.

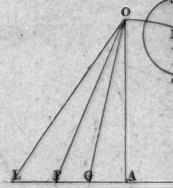

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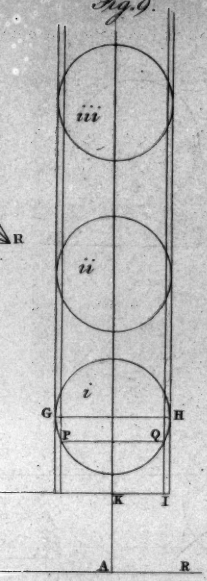
Fig. 9.



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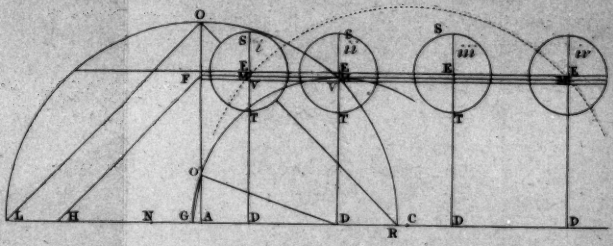


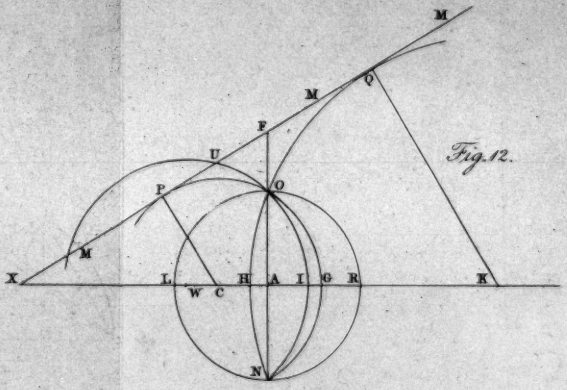
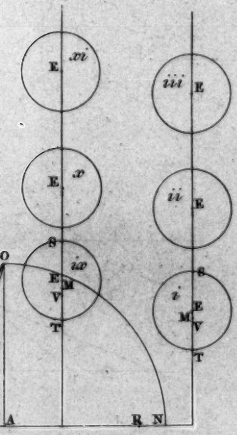


Fig. 12.

Fig. 10.



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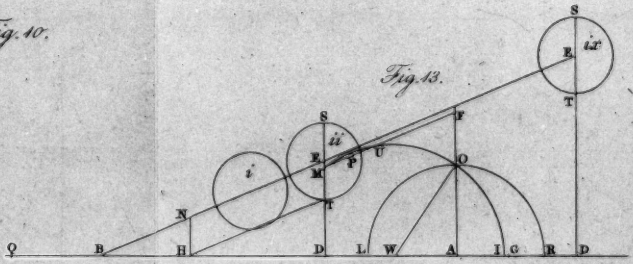


XIX. *On new Properties of light exhibited in the optical Phenomena of Mother of Pearl, and other Bodies to which the superficial structure of that Substance can be communicated.* By David Brewster, *LL. D. F. R. S. Edin. and F. S. A. Edin. In a Letter addressed to the Right Hon. Sir* Joseph Banks, *Bart. K. B. P. R. S.*

Read April 28, 1814.

In the prosecution of my inquiries into the modifications impressed upon light by the various bodies of the animal, the vegetable, and the mineral kingdom, I have had the good fortune to discover several new properties of light, and to establish the laws which regulate the most remarkable of the phenomena. A few of these results have already been laid before the Royal Society; and from the reception which they have experienced, I have taken the liberty of addressing to you the following paper containing a series of new facts, remarkable for their extreme singularity, and their opposition to almost all our notions of the action of bodies upon the luminous rays. The extraordinary character which these experiments at first assumed, induced me to think that I had overlooked some source of deception; and it was not till I had performed them under every variety of circumstances, and till they had been repeated, and viewed in the same light by several of my friends, that I prevailed upon myself to bring them under your notice.

The splendid exhibition of colours which distinguishes

mother of pearl from every other substance, and the successive developement of fresh tints, by every gentle inclination of the plate, have always been ascribed to the lamellated structure of the shell, and have been regarded as a fine proof of NEWTON's theory of the colours of natural bodies. This explanation I had no disposition to call in question, nor did the general train of my experiments lead me to such an inquiry. In examining the coloured rings which mother of pearl, like the topaz, exhibits by polarised light; and in ascertaining the relation between its refractive power, and the angle at which it gives polarity to the reflected ray, I was under the necessity of grinding and polishing, with the utmost care, various plates of this substance. The developement of new colours, and the extinction of others which took place during these processes, indicated the operation of some unknown and extraordinary cause, and encouraged me to pay the most minute attention to all the phenomena which were presented. The results of this investigation I shall endeavour to explain under the four following heads.

I. On the optical properties peculiar to mother of pearl.

II. On the communication of these properties to other bodies.

III. On the causes by which these phenomena are produced.

IV. On a new species of polarisation peculiar to mother of pearl.

I. *On the Optical Properties peculiar to Mother of Pearl.*

Mother of pearl sometimes possesses a regular, and sometimes an irregular structure, and has a striking resemblance to the agate, in the immense variety of forms which it exhibits.

Sometimes it is composed of parallel or concentric laminæ: sometimes the veins are inflected in various successions, and sometimes it exhibits the same appearances as those which constitute what is called the *hammered agate*.

The regularly formed mother of pearl is of an uniform whiteness, somewhat resembling the pearl itself, and, in day light, scarcely exhibiting any of the prismatic colours; and, unless it is expressly mentioned, this is always the kind which I have used in the following experiments.

Let AB Fig. 1. (Pl. XIV.) be a plate of mother of pearl, not polished, but having its two surfaces ground perfectly either upon a blue hone, or upon a plate of glass with the powder of schistus, and let the light R*r* of a candle be incident at any angle on the point *r*, this ray will be reflected according to the ordinary law, so that the angle R*r*C is equal to C*r*S, and the lines R*r*, C*r* and S*r* in the same plane.

If the eye is now placed very close to the mother of pearl at B, so as to receive the reflected rays, it will perceive at S, in the direction *r*S′ the common reflected image of the candle which will not be very bright, owing to the roughness of the reflecting surface. On the lower side of S′ at the distance of some degrees there will also be seen a highly coloured image of the candle at *s*, formed by rays reflected in the direction *rs*. In this spectrum the blue rays are nearest the common image, and the colour is so great, that it requires a prism of flint glass with a refracting angle of 65° to correct it, a large secondary spectrum being left, having the uncorrected green towards the vertex of the prism.

If the candle at R is kept steady, and the plate AB turned round *r* as a centre, so that the ray R*r* may preserve the same

angle of incidence, the coloured ray *rs* will have a motion of rotation about *r*S, the common section AB of the plane *srS* and the surface of the mother of pearl being invariable.

The line AB may be called the *axis of extraordinary reflection*; the extremity A towards which the coloured ray *rs* is reflected, the *primary pole of extraordinary reflection*; *sr*C the *angle of extraordinary reflection*; and *sr*S the *angle of aberration*.

If the ray R*r* is now reflected from the opposite surface of the mother of pearl as represented in Fig. 2. the same phenomena will be observed; but the coloured ray *rs* will now be reflected towards B, and will be seen at *s'* above the common image S', being formed by rays reflected in the direction *rs*. The extremity B therefore of the axis AB will be the primary pole of extraordinary reflection for the lower surface. Hence the two surfaces of mother of pearl have always their poles in opposite directions, unless in specimens where a change of structure takes place.

Let the plate AB be now brought into the position in Fig. 1. where the plane *rs*S coincides with the plane of ordinary reflection R*r*S, and let it be placed upon a goniometer so that we may ascertain by measurement the changes which take place by varying the angle of incidence R*r*C. It will then be found that the angle of aberration *sr*S regularly increases with the angle of incidence. The variations which it undergoes are represented with tolerable accuracy in the following table, but owing to the elongation and indistinctness of the coloured image at large angles of incidence, the measures are not susceptible of great correctness. The first column contains the angle of incidence; the second, the complement of that angle; the third, the angle of aberration as determined by

experiment; and the fourth, formed by adding the 2d and 3d columns, contains the complement of the angle of extraordinary reflection.

Angle of incidence.	Complement of the angle of incidence.	Observed angle of aberration.	Complement of the angle of extraordinary reflection.	Calculated angle of aberration.
86° — 40′	3° — 20′	9° — 14′	12° — 34′	9° — 28′
85	5	8 — 46	13 — 46	8 — 38
82 — 30	7 — 30	7 — 58	15 — 28	7 — 41
80	10	7 — 12	17 — 12	6 — 55
75	15	5 — 50	20 — 50	5 — 52
70	20	5 — 0	25 — 0	4 — 56
65	25	4 — 9	29 — 9	4 — 17
60	30	3 — 45	33 — 45	3 — 44
55	35	3 — 25	38 — 25	3 — 20
50	40	3 — 9	43 — 9	3 — 1
45	45	2 — 53	47 — 53	2 — 48
40	50	2 — 35	52 — 35	2 — 37
35	55	2 — 30	57 — 30	2 — 28
30	60	2 — 25	62 — 25	2 — 21
25	65	2 — 17	67 — 17	2 — 15
20	70	2 — 13	72 — 13	2 — 9
12	78	2 — 7	80 — 7	2 — 7
RrC = SrC	A = SrB = RrA	$x = srS$	A + x = srB	x

If we now compare the angles of aberration in the third column with the angles of extraordinary reflection in the fourth column, and make

A = RrA = SrB the complement of the angle of incidence or ordinary reflection.

$x = srS$ the angle of aberration.

A + x srB the complement of the angle of extraordinary reflection, it will be found that

$$\text{sin. } x : \text{ sin. } x' = \text{sin. } \overline{A' + x'} : \text{ sin. } \overline{A + x}$$

That is, the sines of the angles of aberration are to one another inversely, as the sines of the complements of the angles of extraordinary reflection.

Assuming the numbers in columns 3d and 4th, I have upon this principle computed those in the 5th, which are the calculated angles of aberration, and which agree very strikingly with the observed angles.

If we now turn round the mother of pearl 180° so that the pole A may be brought into the position of B, the ray *rs* will be reflected towards the pole A, fig. 2. and the complement of the angle of extraordinary reflection will be equal to the difference between the angle of aberration and the complement of the angle of ordinary reflection. In this case we shall obtain the results given in the following table.

Angles of incidence	Complement of the angle of incidence.	Observed angle of aberration.	Complement of the angle of extraordinary reflection.	Calculated angle of aberration.
60°	30°	4° — 30′	25° — 30′	4° — 33′
55	35	4 — 37	31 — 23	3 — 46
50	40	3 — 11	36 — 49	3 — 16
45	45	2 — 57	42 — 3	2 — 56
40	50	2 — 38	47 — 22	2 — 40
30	60	2 — 16	57 — 44	2 — 19
20	70	2 — 7	67 — 53	2 — 7
	A	*x*	A — *x*	*x*

By comparing the observed angles of aberration with the complements of the angles of extraordinary reflection, we shall find that

$$\text{sin. } x : \text{sin. } x' = \text{sin. } A' - x' : \text{sin. } A - x$$

which indicates the same relation as formerly between the angles of aberration and extraordinary reflection. Upon this principle I have computed the numbers in the 5th column which agree very well with the observed angles.

While the primary pole A is describing a semicircle round *r* till it reaches B, and another semicircle from B to A again,

a point *s* in the extraordinary ray *rs* will describe a curve round S composed of two semiellipses having the same conjugate axis, but having their semitransverse axes of different lengths. Thus, in fig. 3, if S be a point in the reflected ray *r*S (fig. 2), and S*s*, SN, S*t*, SM, different values of the angles of aberration when the pole A is in the directions S*s*, SN, &c.; then the point *s* will describe the semiellipse M*s*N, and the semiellipse M*t*N which have their conjugate axis MN common, and their semitransverse axes S*s*, S*t* of unequal lengths. The conjugate axis MN is a constant quantity, while S*s*, S*t* vary according to the law already mentioned. At different angles of incidence, therefore, the point *s* will describe other curves such as M*s'*N*t*, which approach to a circle as the angle of incidence diminishes.

The angles of aberration vary in different pieces of mother of pearl, but there is no deviation from the laws which have just been explained.

On the outside of the extraordinary ray *rs*, fig. 1, a mass of coloured light *rp* makes its appearance nearly at the same distance from the extraordinary image, that the extraordinary image is from the common image: these three images are always in a straight line, but the angle of aberration of the mass of coloured light varies according to a law different from that of the extraordinary ray. At great angles of incidence, this mass of light is of a beautiful crimson colour. At an angle of about 37° it becomes green, and at less angles of incidence it acquires a yellow hue, approaching to white, and becomes very luminous. These colours which become more brilliant when the mother of pearl is polished, vary with the thickness of the plate.

The fracture of mother of pearl always reflects the extraordinary ray *rs*, as if the surface of the fracture were parallel to the real surface; but when the fracture is ground flat, no extraordinary reflection takes place.

When the extraordinary ray *rs* is reflected from another piece of mother of pearl, it experiences, as might have been expected, both an ordinary and an extraordinary reflection. In virtue of the ordinary reflection, an image is formed exactly like the extraordinary image, but in virtue of the extraordinary reflection the highly coloured image is sometimes rendered more highly coloured, and at other times converted into a greenish white image, according as the second reflection conspires with or opposes the first.

Hitherto we have attended to the phenomena only when the surface is rough and unpolished. When a slight degree of polish, however, is communicated to it, a new coloured image appears on the opposite side of the common image formed by the rays *rt*, fig. 1 and 2. This new image resembles in every respect the other coloured image, and follows the same laws; and after a high degree of polish is induced upon the mother of pearl, it is almost as bright as the first coloured image which has its brilliancy somewhat impaired by polishing. If the polish is removed by grinding, the second coloured image vanishes, and the first resumes its former brilliancy. As this second image is reflected towards the pole A in fig. 1. and the pole B in fig. 2, they may be called the *secondary poles* of extraordinary reflection.

If we now examine the light transmitted by the mother of pearl, we shall perceive phenomena analogous to those which have been described. A coloured image will appear on each

side of the common image, having the same angles of aberration as those seen by reflection, and resembling them in every respect, the blue light being nearest the common image, and the red light farthest from it. These two images, however, are usually fainter than those seen by reflection, and when the second extraordinary reflected image is extinguished by removing the polish, it is then the most brilliant when seen by transmission, and in general the image which is brightest by reflection is faintest by transmission.

In some irregularly formed pieces of mother of pearl which are ground very thin, and in which the axes of extraordinary reflection for the two surfaces are not coincident, four coloured images are seen by transmission. Two of them are produced by each surface, and the line which joins the two images formed by the same surface, always coincides with its axis of extraordinary reflection. It is also deserving of notice, that the transmitted extraordinary ray is bent towards the same pole as the extraordinary reflected ray to which it belongs.

Like all other bodies, mother of pearl polarises light by reflection, and the angle at which the quantity of polarised light is a maximum, is about 59°. The two extraordinary images are also polarised at the same angle, but the mass of green and crimson light exhibits no marks of polarity. This substance has also the property of depolarising light in every position like horn, tortoise shell, caoutchouc, and gum Arabic; and it exhibits by polarised light those brilliantly coloured rings, which I have described in a former paper, as produced by topaz and other bodies.

II. *On the communication of the properties of Mother of Pearl to other bodies.*

The phenomena which have now been described must be admitted to be very singular and instructive, even if the inquiry had been pushed no farther; and it is probable that philosophers would have contented themselves with ascribing them to reflections from differently inclined planes in the interior of the mother of pearl.

In order to measure the angles contained in the preceding tables, I had occasion to fix the mother of pearl to a goniometer by a hard cement. Upon removing it from the cement, the plate left a clean impression of its own surface, and I was surprised to observe that the cement had by this means received the property of producing the colours which were exhibited by the mother of pearl. This strange result I at first attributed to a thin film detached from the plate, but subsequent experiments soon convinced me that this was a mistake, and that the mother of pearl really communicated to the cement the properties which it possessed.

I have also succeeded in imparting the same faculty of producing colour to black and red wax, balsam of Tolu, gum Arabic, gold leaf placed upon wax, tinfoil, the fusible metal composed of bismuth and mercury, and to lead by hard pressure, or by the blow of a hammer. When the impression is first made upon the fusible metal, the play of colours is singularly fine, but the action of the air corrodes the metal, and speedily destroys the configuration, as well as the polish of its surface. The same effect was produced when the metal was immersed in oil.

In order to shew that in these cases no part of the mother of pearl is detached, I plunged the wax, after it had received the impression, into nitric acid, which had no effect either in destroying or diminishing the colorific property of the surface. In soft cements made of bees' wax and rosin, the slightest degree of heat destroyed the superficial configuration by which the colour is produced. In sealing wax, gum Arabic and realgar, a much greater heat was requisite to remove the colour; and in tinfoil and lead this could only be effected by the temperature at which they cease to become solid.

Let us now examine more minutely the phenomena which present themselves when the light is reflected from the surface of wax; and let us suppose that the impression upon the wax is made by the lower surface, when rough, of the mother of pearl, as represented in fig. 2, where B is its primary, and A its secondary pole. When the light R*r* of a candle is reflected from the surface of wax AB, fig. 3, the extraordinary image, instead of being reflected towards the primary pole B as in fig. 2, is reflected from it, and A is the primary pole of the wax, whereas B was the primary pole of the mother of pearl. By polishing the mother of pearl, and taking a new impression from it, the wax will now reflect the other extraordinary image in the direction *rt*, and therefore B is the secondary pole of the wax. Hence it follows that *mother of pearl communicates to wax and other bodies the optical properties of the surface opposite to that from which the impression is taken.*

At different angles of incidence the two coloured images formed by the wax follow the same laws as those produced by the mother of pearl; but the mass of green and crimson

light never appears, and is therefore caused by some internal structure which cannot be communicated to other bodies. When an impression is taken from the fracture of mother of pearl, its faculty of producing colour is also communicated.

In imparting to gum Arabic and balsam of Tolu, the superficial configuration of mother of pearl, we are enabled, on account of their transparency, to observe the changes induced upon the transmitted light. The extraordinary images formed by reflection were both visible, the primary one being remarkably brilliant, and the secondary one scarcely perceptible; but when the light was transmitted through the gum the primary image was nearly extinct, while the secondary one was unusually brilliant and highly coloured, far surpassing in splendour those which are formed by transmission through the mother of pearl itself. When both the surfaces of gum Arabic are impressed with mother of pearl, four images are seen. The colours seen by transmission are more brilliant in the gum than in the balsam, as the latter has the greatest reflective power; but the coloured images produced by reflection do not seem to have suffered a greater dispersion, when they are formed by the metals, than when they are formed by cements.

When the impression is taken from a pearl, the wax receives a character similar to that which is possessed by the pearl. The image reflected from the surface of the pearl is enveloped in a quantity of unformed light, arising from a cause which will afterwards be explained; and the very same white nebulosity is reflected from the wax.

III. *On the causes of the Phenomena of Mother of Pearl.*

From a careful examination of the preceding facts, we must now be prepared to infer, that all the peculiar phenomena of mother of pearl, as seen by reflection and transmission, are *owing to a particular configuration of surface; that the communication of these properties to other bodies, is the necessary consequence of the communication of its superficial structure; and that none of the light which is concerned in the production of these phenomena, has penetrated the surface of the mother of pearl.*

What this configuration of surface is, and in what manner it generates the coloured images, are points of high interest, and of corresponding difficulty. The facts naturally lead us to conjecture, that the extraordinary reflections are produced by faces, either curved or rectilineal, slightly inclined to the general surface of the mother of pearl. In attempting to determine this point, I anticipated no assistance from microscopical observations, as it was contrary to all our notions of the action of bodies upon light, to imagine that a plate of mother of pearl reflecting an image as perfectly as the mirror of a telescope, could exhibit to the human eye any superficial irregularities. These anticipations however were wholly erroneous. By the application of single microscopes with powers of 200, 300, and even 400, I have discovered in almost every specimen of mother of pearl, an elementary grooved surface, which no polishing can modify or remove. This structure resembles very closely the delicate texture of the skin at the top of an infant's finger; or the lines parallel to the coast upon a map, by which the engraver marks the limits of the sea and land. When the mother of pearl has a regular

structure, these grooves or lines are always parallel, but when there is any irregularity of configuration, the grooves vary their direction, and are arranged in all possible forms like the veins of agate, or like the lines upon the coast of a map, where there are numerous inlets and islands to be represented.

Sometimes the spaces between the grooves are so wide, that they can be seen with a magnifying power of six or eight times, and in one or two specimens I have observed them with the naked eye. At some parts of the surface the distance between the grooves is so small, that I have counted more than 9000 in an inch; and in some pieces they can scarcely be detected with any magnifying power which I have been able to apply. When the space between the grooves is large, a new groove often commences, and there is frequently a sudden change from a space with a series of distant grooves, to another space with a series of very close ones. Similar appearances were also seen in the structure of pearls. When the mother of pearl is scratched or indented, the bottom and the sides of the scratch are grooved exactly like the parts that are polished. The same grooved structure is likewise distinctly seen in wax, gum Arabic, and the metals, after they have received the impression of the mother of pearl.

In every case the grooves are at right angles to the axis of extraordinary reflection, and hence in irregularly formed mother of pearl, where the grooves are often circular, and have every possible direction, the axes of extraordinary reflection have also every possible direction, and the coloured images appear irregularly scattered round the ordinary image. In the real pearl, these coloured images are crowded into a small space round the common image, on account of the spherical form of

the pearl, and the various hues are thus blended into a white unformed light, which gives to this substance its high value as an ornament.

Having thus ascertained that the surface of mother of pearl is actually grooved, and different in this respect from all other bodies that have yet been examined, we must now seek, in this peculiarity of structure, the cause of its optical properties. The facts which have been detailed in the beginning of this letter will enable us to draw several important conclusions of a general nature, but they leave us in the dark respecting the immediate cause of the phenomena.

Let us now suppose that fig. 5. represents a section of a plate of mother of pearl, having *AanmbcB* for its upper surface, and *B'a'n'm'b'c'A'* for its lower surface. Let OP be the line at which the attracting or refractive force ends, and where the repulsive or reflecting force begins, and let the reflecting force terminate at MN according to the NEWTONIAN theory. We have already seen that when the surface AB of the mother of pearl is ground as flat as possible, and brought to a high polish, the light which is incident on the repulsive stratum MNOP is reflected as in all other bodies, and affords a perfect image of the object from which it radiates. Hence it follows that the light which forms the extraordinary images has escaped reflection, and penetrated the attractive stratum OP; and that its separation into colours and extraordinary reflection are produced by one or more causes residing between OP and the surface AB of the mother of pearl.

Let us first attend to the aberration of the extraordinary images. Since the real surface of AB is composed of faces inclined to the general surface A*b*cB, we are led to suppose

that the primary extraordinary image is reflected from the face m, while the secondary extraordinary image is reflected from the face n. Now this could only happen from two causes, either in consequence of the mother of pearl having a repulsive force different from the ordinary repulsive force which produces reflection; or from its possessing the power of reflecting light from its actual surface. That this extraordinary force is, in other respects like the ordinary reflecting force, is manifest from a portion of the extraordinary pencil being transmitted, while the other portion suffers reflection. The existence of such a force being unquestionable, we have next to consider the form and position of the surfaces to which it belongs. The changes in the angle of aberration at different angles of incidence, is a proof that the surfaces m, n, present different inclinations to the incident ray; and hence their form must be curvilineal, as represented by the dotted lines above m and n. If we suppose that the ray has been refracted before it experiences the extraordinary reflection, the angles of aberration still require that the faces have a curved form.

Taking the index of refraction $m = 1.653$ and making $A =$ angle of incidence; $a =$ angle of refraction, and consequently the new angle of incidence upon the faces m, or n; $b =$ angle of extraordinary reflection, and $x =$ the inclination of the reflecting face m or n, then we shall have

$$\text{sin. } a = \frac{\text{sin. } A}{m} \text{ and } x = \frac{a+b}{2} - a = \frac{b-a}{2}$$

By calculating the value of x for different angles of incidence, we obtain the following results:

Angles of incidence.	Inclination of the faces *m* or *n*.
86° 41′	20° 8′
70	15 9
65	13 48
60	12 19
12	1 20

But since the refraction cannot be completed when the ray reaches the surfaces *m* or *n*, having passed through only half the space of refracting activity, let us suppose the value of *m* corresponding to the partial refraction to be 1.300, and we shall have the following values of *z*:

Angles of incidence.	Inclination of the faces *m*, *n*.
86° 41′	18° 34′
70	9 16
65	8 19
60	7 14
12	0 20

If the light, therefore, suffers either a total or a partial refraction, or if it suffers no refraction at all, the extraordinary reflection must be made from faces of variable curvature.

The most unaccountable circumstance, however, accompanying the extraordinary reflection, is the difference of effect produced upon the surfaces *m* and *n* by removing the polish. The surface *m* retains its power of reflecting the primary extraordinary image, notwithstanding the roughness which is thus superinduced, while the surface *n* loses the power of reflecting the secondary image, and acquires the faculty of transmitting the whole of the coloured pencil which composes it. The force therefore which reflects the primary extraordinary image would appear to be different from that which reflects the secondary extraordinary image, the latter being wholly dependant on the smoothness of the surface.

Hitherto we have avoided all consideration of the cause which separates the extraordinary pencils into their component colours, nor can we pretend to afford even a plausible conjecture respecting their origin. It is quite obvious that the separation into colours is produced before the pencil suffers extraordinary reflection, and as the transmitted colours are not complementary to those which are reflected, it is equally manifest that the phenomenon has no connection with the colours of thin plates. If the spectra were produced by the ordinary dispersive force of the body, then the dispersion ought to be least in gum Arabic, greater in wax, and still greater in realgar and the metals, whereas in all these cases the quantity of colour appears to be the same. The extraordinary spectra have no resemblance whatever to those which are the effect of inflexion; and even if we could suppose that the light was inflected by the grooves, the cause would be inadequate to explain the continuance of the colour when the plane of incidence coincides with the direction of the grooves, and when there are manifestly no angles to bend the passing light.

But whatever be the cause of the phenomena of mother of pearl, the facts themselves are peculiarly instructive, and naturally lead us to the following conclusions.

1. Besides the ordinary forces which reflect and refract light, there reside without the surface of mother of pearl, and of all bodies to which its superficial configuration can be imparted, new forces which reflect light, and separate it into its component colours.

2. The lines which bound the space of reflecting activity in all surfaces which possess this configuration, are straight, and are not parallel to the grooved structure of the surface. Hence a surface which appears, even to the unassisted eye, to be full

of eminences and depressions, is capable of reflecting light with perfect accuracy.

3. Since a particular configuration of surface, independent of chemical composition and crystalline structure, is capable of producing the most brilliant colours, may not the colours of all natural bodies be owing to the arrangement of their superficial particles; and may not the changes which these colours undergo by the action of light, heat, and atmospherical causes, arise from a corresponding change in their superficial structure?—I have endeavoured to communicate to wax the faculty of producing colour possessed by Labrador spar, the metallic oxides, and various other bodies; but though I have not succeeded in this attempt, it by no means follows that the colour is not produced by the configuration of the surface. The structure may in these cases be so minute, that fluid wax cannot be forced into the grooves or depressions; and we have an approach to this delicacy of conformation in some specimens of mother of pearl, where the grooves cannot be seen by the most powerful microscopes.

4. Since a particular structure of surface is always accompanied with a new repulsive force, residing nearer the body than the common repulsive force which produces ordinary reflection, may there not reside also, near the surface of all crystallised bodies, a new refractive force which produces double refraction? And is not this supposition countenanced by the fact, that the extraordinary pencil formed by Iceland spar suffers the ordinary as well as the extraordinary refraction?

I shall now conclude this section with a few remarks on the crimson and green light which always accompanies the primary coloured image. This mass of light is never produced by the wax, and as it appears, even when the rays are

incident upon the mother of pearl from a fluid of the same refractive power, it is evidently unconnected with the form of surface. These masses of colour appear to have the same origin as the colours of thin plates described by NEWTON. Even when the angle of incidence is the same, the crimson light appears at one thickness of the mother of pearl and the green at a less thickness, and the transmitted light consists of colours complementary to those of the reflected light. We are therefore, in this case, presented with phenomena almost exactly the same as those of thin plates, though produced by plates of mother of pearl of considerable thickness;—a subject which is well deserving of more attentive examination.

IV. *On a new Species of polarisation peculiar to Mother of Pearl.*

Having seen, in the course of the preceding experiments, so many deviations from the ordinary laws of optics, I suspected that mother of pearl might exhibit similar anomalies in the polarisation of light. This conjecture was immediately confirmed by the discovery of a remarkable property, which forms the connecting link between the phenomena of polarisation, as effected by crystallised and uncrystallised bodies.

In all doubly refracting crystals the opposite polarisation of the two images is invariably related to some axis or fixed line in the primitive form; while in all uncrystallised bodies the polarisation is related to the planes of reflection and refraction, the reflected pencil being always polarised in an opposite manner to the refracting pencil. Thus, if AB, fig. 6, be a plate of glass, and R*r* a ray incident upon it at the polarising angle, the reflected ray *r*S will be polarised in the same manner as one of the pencils formed by calcareous spar, and a small portion of the transmitted ray *r*T will also be polarised,

but in a manner opposite to *r*S like the other pencil in calcareous spar: or if the ray R*r* is transmitted through a bundle of glass plates, the whole of the pencil *r*Γ will be polarised in that manner.

If we now suppose AB a single plate of mother of pearl about one-fortieth of an inch thick, and the angle of incidence R*r*C about 60°, the reflected ray *r*S will be polarised as in every other transparent body; but the transmitted ray *r*T *will be wholly polarised, and in the same manner as the reflected ray* *r*S, while in every other transparent body that has been examined the ray *r*T possesses an opposite kind of polarisation. If we turn the plate AB round its centre *r*, so as to preserve its inclination to the incident ray R*r*, no change whatever takes place, the transmitted ray still retaining its former polarity.

The angle of incidence R*r*C, at which the transmitted light *r*T is wholly polarised, varies in the inverse ratio of the thickness of the plate AB, and the whole pencil is polarised at any angle greater than that angle. The relation between the angle of polarisation and the thickness of the plate remains to be determined; though I suspect it will be found that the tangents of the angles of incidence at which the whole of the pencil is polarised, are inversely as the thickness of the plates.

The phenomena which I have now described, I have observed in every piece of mother of pearl that I have tried; and as they are not affected when the incident pencil is refracted from balsam of Tolu, or any other cement, into the mother of pearl, they are obviously unconnected with its superficial configuration. Ivory does not produce the same effect upon light.

From these results the following conclusions are clearly deducible.

1. That mother of pearl polarises light in a manner different from all *crystallised bodies*, the polarisation having no reference to any fixed line in the plate.

2. That mother of pearl polarises light in a manner different from all *uncrystallised bodies*, the transmitted pencil being wholly polarised by a single plate, and in the same manner with the reflected pencil.

3. That if mother of pearl polarises light in virtue of its laminated structure, the laminæ themselves must have the property of polarising light in a manner opposite to all other bodies.

I have thus endeavoured to explain, as briefly as possible, the various phenomena exhibited by mother of pearl in its action upon light. The subject, however, is far from being exhausted: it still presents many points of interesting inquiry; and if the investigation could be carried on with the aid of analogous phenomena, we might confidently look forward to some great change in the fundamental principles of physical optics.*

I have the honour to be, Sir,
your most obedient and humble servant,
DAVID BREWSTER.

Edinburgh, February 28th, 1814.

To the Right Hon. Sir JOSEPH BANKS, Bart. K. B. P. R. S. &c. &c. &c.

* Since this paper was written I have made a variety of additional experiments on the superficial and communicable colours of Mother of Pearl and other bodies. An account of these experiments, and an explanation of the phenomena contained in Sect. IV. of the preceding letter, will form the subject of another communication.

Fig. 1.

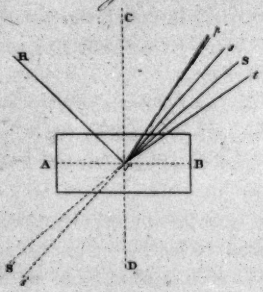


Fig. 2.

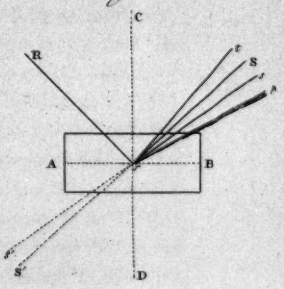


Fig. 3.

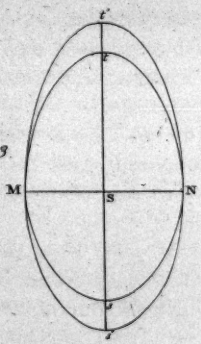


Fig. 4.

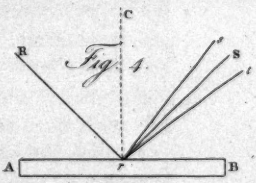


Fig. 5.

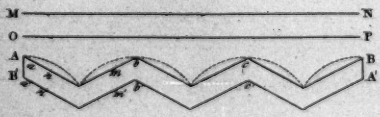
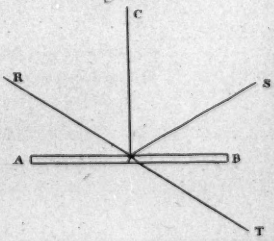


Fig. 6.



XX. *An improved method of dividing Astronomical Circles and other Instruments.* *By Capt.* Henry Kater. *Communicated by* Thomas Young, *M. D. For. Sec. R. S.*

Read May 5, 1814.

THE art of dividing astronomical circles and other instruments for taking angles, has ever been thought highly important to the improvement of astronomy and geography, and has consequently, at different periods, formed a subject of much attention and inquiry. The method of dividing, which until very lately was in general use, was that practised so successfully by the late Mr. BIRD; but the laborious and delicate operation by the scale and beam compasses, has recently been superseded by the invention of Mr. TROUGHTON, of the excellency of which the mural circle at the Royal Observatory at Greenwich affords a noble proof. But this method, though decidedly superior to Mr. BIRD's, seems yet to leave somewhat more to be done with respect to simplicity and facility of execution; and it is with the hope of having in some measure attained these desirable objects, that I am induced to lay before the Royal Society a mode of dividing, which, I am led to believe, will not be found inferior in accuracy to any that has hitherto been proposed.

In the essential principle on which this method is founded, I find that I have been anticipated by the Duc de CHAULNES; but some parts of the apparatus which he has described are so

complicated, and others so incapable of the necessary adjustments, that his method of dividing never has been, nor ever can be introduced into practice, and indeed was intended by its author merely to form a model, which was to be employed as part of a dividing engine.

The method which I am about to describe requires no other apparatus than two pieces easily made, which I shall call *adjustable dots;* three microscopes, (two furnished with micrometers) and a cutting point with its frame.

The general principle of this method consists in viewing the two micrometer microscopes, as the points of a pair of beam compasses, and the moveable dots as divisions, capable of the nicest adjustment previously to their being transferred to the limb of the instrument. Its principal advantages are its simplicity, and the exclusion of all errors, excepting such as may arise from careless execution, the imperfection of vision, or the instability of the apparatus employed.

The *adjustable dots* are represented at plate XV. figs. 1, 2, and 3. *a*, *a*, *a*, *a*, fig. 1. is a flat piece of brass, intended to rest on the face of the arc to be divided, and to be clamped to it by means of the screw C, fig. 3, where the adjustable dot is represented in perspective. The piece *c c*, slides smoothly in a dove-tail groove, formed by the pieces *b*, *b*, and is moveable by the screw *d*. *e*, *f*, slides in like manner in a dove-tail groove beneath, and at right angles to the piece *c c*, and is moveable by the screw *g*. That part of the clamp which bears against the edge of the arc, should be curved in such a manner as to have only its two extremes in contact with the edge, by which means the motion of the sliding piece *c c*, will always be in the proper direction. A very thin and narrow tongue

of silver or gold, on which is a well defined circular dot, is soldered to the lower surface of *ef*, beneath, and projecting from *e*: the lower surfaces of this tongue, of the piece *ef*, and of *a*, *a*, *a*, *a*, are all in the same plane. It is evident from this construction, that the tongue with the circular dot is moveable on the surface of the limb of the instrument, both to and from its centre, and in a direction at right angles to the radius. The screws *d*, and *g*, are to be turned by a key with a handle of wood or ivory, in order to avoid any change of temperature which might arise from the near approach of the hand; the sliding pieces should move easily, and yet be perfectly free from shake.

There is another mode of constructing the adjustable pieces, which I conceive may be found far preferable to that I have just described, though in the use of them greater care and nicety will be required. It is to draw two fine lines at right angles to each other on the silver tongue of the piece *ef*, fig. 1. which lines may be used instead of the dot, by causing them to bisect the angles formed by the cross wires of the microscopes, and thus ensuring, as far as vision is concerned, as great a degree of accuracy in dividing as can afterwards be attained in reading off.

The line in the direction of the piece *ef*, must be drawn when the adjustable piece is fixed on a circle, and when the piece *cc*, is placed about half way between the limits of its adjustment; in which position this line will be directed towards the centre of the instrument, and, unless the circle to be divided is very small, the difference from this direction within the limits of its adjustments will scarcely become sensible. This will be better understood when the manner of using the

adjustable dots has been described. The silver tongue with its cross lines is represented at A, plate XV. fig. 1.

The three microscopes are similar to those used in reading off the divisions of astronomical instruments of modern construction, and therefore need not be very particularly described. Two of these microscopes are furnished with micrometers in the usual manner, each having a set of cross wires as represented at fig. 6, moveable in the direction *ab*, by means of the micrometer screw.

The two micrometer microscopes have adjustable supports, one of which is seen at fig. 4. in which *a*, *a*, are two pieces of brass forming a dovetail groove for the sliding piece A, which carries the microscope, and which is moveable by the screw represented in the plate. In the circular opening formed in the sliding piece A, a piece of brass tube is soldered, in which the microscope slides rather stiffly, in order to be adjusted to distinct vision. The supports are made very strong, and are furnished beneath with clamps, by which, or by any other equivalent contrivance, they may be firmly fixed to an arc, hereafter to be described. One of the microscopes with its support is represented at fig. 5, from which its construction will be readily understood; the micrometer head is omitted in the engraving, as it would only occasion confusion. The object glass of each microscope is fixed in a tube having a fine screw cut on its outside, by means of which, after the microscope has been adjusted as nearly as possible to distinct vision, any remaining parallax may be removed, when the object glass is to be secured by the nut *a*, in the usual manner.

Fig. 7, represents the support for the third microscope, and also the cutting point, the frame of which is attached to this

support. ABCDE, is a thick plate of brass, having a tube soldered in the opening C, to receive the microscope. The projecting square pieces B, D, receive the conical points of the screws F, F, of the cutting frame, the general construction of which will be readily understood from the engraving. The cutting point which is elliptical, as recommended by Mr. TROUGHTON, passes obliquely at an angle of about fifty degrees through the flat piece of brass G. The pieces *a*, *a*, project half an inch or more beyond the piece G, and to their under surfaces a thin slip of silver or gold is soldered, or otherwise securely fastened, on which a well defined circular dot is made, as represented in the plate. The sides HI, HI, of the cutting frame, must be of a sufficient length for the ends I, I, to reach nearly to the surface of the instrument to be divided when the frame hangs perpendicularly from B, D; by this construction of the cutting frame, it is evident that the circular dot may be adjusted so that its image may pass through the point of intersection of the wires of the microscope when carried through its field of view, and that the cutting point may be seen through the same microscope when employed in tracing the divisions on the limb of the instrument. A profile of the cutting point and frame is given at fig. 8.

As every thing depends on the rectilineal motion of the cutting point when tracing a division, too much care cannot be employed in its use, and it is greatly to be wished some better mode of giving it motion could be adopted in place of the inflexible projecting handles at present employed, as a lateral deviation of only $\frac{1}{10000}$ of an inch, in a circle of two feet diameter, would occasion an error in the division amounting to about a whole second. Perhaps an universal joint applied to

the upper surface of the piece G, and connected with a handle, might be used with advantage. The length of that part of the cutting point which projects beneath the piece G, may also prove a source of error; but this may be easily obviated by making the piece G, much thicker, and passing the screw through it, as represented in the profile.

An arc of cast iron or brass must be provided, of about 120 degrees in length, and of a radius rather greater than that of the instrument to be divided. Its breadth may be about two inches, and its thickness should be fully sufficient to ensure its not bending. This arc, which is designed to carry the three microscopes, is to be fastened at its extremities by clamps, screws, or otherwise, to the framework of the instrument to be divided, in such a manner as that the microscopes may project over its limb, and be at a proper distance above it for distinct vision, and that the surface of the arc may be parallel to the face of the instrument. The manner of fastening this arc must be left to the ingenuity of the artist. In some instances it may perhaps be found convenient to construct a temporary frame for the reception of the instrument during the operation of dividing, but in whatever manner the arc is applied, it is absolutely necessary that it should be so secured as to be perfectly immoveable.

I shall now proceed to detail the manner in which this apparatus is to be employed in dividing a circle, which I shall suppose to be two feet in diameter. The first operation necessary is to describe on the face of the instrument the circles which are to limit the divisions. This may readily be done by causing a fixed point to press on the surface of the circle, whilst it is made to revolve on its axis. The support carrying

the third microscope and cutting frame, which microscope I shall in future distinguish by calling it *the fixed microscope*, is then to be firmly screwed to the arc, as near to its right extremity as conveniently may be, and so that the intersection of the cross wires of the microscope may be a little without the exterior circle described on the face of the instrument, and yet that both the circles that limit the divisions may pass through the field of view. The support of the fixed microscope must also be so placed, as that the cutting point may move in the direction of the diameter of the circle to be divided. The moveable microscopes are to be attached to the arc by their clamps, and to be always so adjusted that the intersections of their wires may appear half way between the circles intended to limit the divisions. The microscope which is placed next to the fixed microscope I shall distinguish by the letter A, and the other by the letter B. The micrometer head of A must be placed to the right, and that of B to the left, of their respective microscopes.

It will, perhaps, be found most convenient to commence by dividing the circle into five equal parts of *seventy-two degrees* each. For this purpose, having fixed the circle by means of the clamp attached to its tangent screw (with which every instrument is provided), draw a fine line with the cutting point from the exterior circle described on the face of the instrument to the edge of its limb: this line I shall call *the line of reference*. Adjust the fixed microscope to distinct vision, by sliding it in its support, and cause the line of reference accurately to bisect the vertical angles formed by the cross wires of the microscope, by moving the circle, and by turning the microscope in the tube of its support. Move the eye to the right and left, and observe

whether the line of reference appears to change its position. If it is perfectly stationary in every situation of the eye, the adjustment of the microscope is correct; if not, the remaining parallax must be destroyed by moving the object glass, which must afterwards be secured by means of its nut.

The next step is to adjust the cutting frame. To this end, place the projecting pieces *a*, *a*, of the cutting frame on the face of the limb of the instrument, when the dot will be at a proper distance from the microscope for distinct vision. Slide it through the field of view, and examine whether its centre passes accurately through the intersection of the cross wires, and if not, cause it to do so by means of the screws F and F.* This adjustment should be most carefully examined in every part of the process of dividing, as it is essentially necessary to the accuracy of the operation.

Place the microscope A, as near to the fixed microscope as conveniently may be (leaving, however, sufficient space to use the cutting point), and clamp it firmly to the arc. Fix the microscope B at the distance of seventy-two degrees by estimation from A,† and bring the intersections of their wires between the circles which are to limit the divisions. Place an adjustable dot under A, with its centre precisely in the intersection of the cross wires. Unclamp the circle, and turn it on its axis till the adjustable dot arrives at the intersection of the cross wires of B; and should the centre of the dot not pass

* The same thing might, perhaps, be as conveniently effected by giving a motion to the cross wires of the fixed microscope, and fine lines intersecting each other at right angles may be used instead of the dot (as in the adjustable pieces) should it be thought preferable.

† This may be readily done by means of the figures usually engraved on the limb previous to the operation of dividing.

accurately through their intersection, cause it to do so by means of the screw of the moveable support of the microscope B. The intersections of the cross wires of both microscopes will now be equidistant from the centre of the circle. Adjust both microscopes to distinct vision, in the manner described for the fixed microscope, using the adjustable dot instead of the line of reference. Return the line of reference to the fixed microscope, clamp the circle, and, by means of the tangent screw, bring the line of reference again accurately under the intersection of the cross wires. Place the second adjustable dot precisely under the intersection of the wires of the microscope B. When all has been thus arranged, the line of reference, and the two adjustable dots, should appear in the intersection of the wires of their respective microscopes, and the dot of the cutting frame, if moved through the field of view, should pass through the intersection of the wires of the fixed microscope. This arrangement of the microscopes is represented in plate XVI. from which the manner of placing them in the subsequent parts of the operation will also be readily understood.

Turn the circle on its axis till the dot which was under A, arrives under B. Clamp the circle, and, by means of the tangent screw, bring the centre of this dot under the intersection of the wires of the microscope B. Take off the adjusttable dot which was under B, and which will now be found to the left of that microscope, place it beneath A,* and, by means of its adjusting screws, bring its centre under the intersection of the cross wires, carefully remarking whether the circle has

* The artist will perceive, that five dots in this stage of the process will be more convenient than two, and obviate the necessity of removing the dots.

in the mean time suffered any change of position, which may be known by the centre of the dot which is under B, being no longer found precisely under the intersection of its wires; and, should this have happened, the dot must be restored to its former position by moving the circle, previous to the final adjustment of the dot which is under the microscope A. The adjustable dots must thus be made successively to change places, till the line of reference arrives again in the field of the fixed microscope, when it is evident, if the intersections of the wires of the microscopes A and B are precisely distant seventy two degrees from each other, the line of reference will again be found accurately in the intersection of the cross wires of the fixed microscope, as it was at the commencement; but should this not be the case, its distance from it will be equal to five times the error in the distance of the wires of the microscopes A and B, and this error is to be corrected by the micrometer of the microscope A,* and the whole operation repeated, always remembering to bring, in the first instance, the line of reference accurately to the intersection of the wires of the fixed microscope, and carefully to repeat the various adjustments which have been detailed.

When, by repeated trials, the wires of the microscopes A and B are found to be at the required distance from each other, the divisions are to be cut on the limb of the instrument, the eye being assisted by viewing the cutting point through the fixed microscope. The first line, or zero, is to be made when

* The line of reference may be brought to the intersection of the wires of the fixed microscope and the distance of the dot which is under A, from the intersection of its wires be measured by its micrometer screw, and this will be equal to five times the error in the distance of A and B.

the line of reference and the two dots are under the intersections of the wires of their respective microscopes, and each of the other divisions, successively after the dots have been made to change their places; and thus the circle will be divided into five equal parts, which seem to be subject to no other error than what may be attributed to the imperfection of vision, or inaccurate execution.

Each division of *seventy-two degrees* is next to be subdivided into three parts. To effect this, bring the line of reference to the intersection of the wires of the fixed microscope, and verify the adjustment of the cutting frame. Place the microscope B, above the first division from the fixed microscope, and the microscope A, at the distance of one third of a division, or twenty-four degrees by estimation from B. Cause the division which is under B, to bisect the vertical angles of the cross wires, by means of the micrometer screw, and by turning the microscope in its tube, and adjust both microscopes to distinct vision. Place an adjustable dot under the intersection of the wires of A; unclamp the circle, and turn it till the adjustable dot is under B; when, if it does not pass precisely through the intersection of the cross wires, it must be made to do so, as before described, by means of the screw attached to the support of the microscope. Bring back the line of reference to the wires of the fixed microscope, when the adjustable dot should be precisely in the intersection of the wires of A, and the first division (seventy-two degrees from zero) accurately bisecting the vertical angles of the cross wires of B. Turn the circle round till the adjustable dot arrives under B, and by means of the tangent screw bring it accurately to the intersection of the cross wires. Place another adjustable

dot in the intersection of the cross wires of A, taking care, by examining the dot under B, to ascertain that the circle has not in the mean time suffered any change of position. Bring the dot which was last placed, and which is now under A, to the intersection of the wires of B, when it is evident if the wires of the microscopes A and B, are at the required distance from each other, the next division of the instrument, or zero, will now pass precisely through the intersection of the cross wires of A. But should this not be the case, the distance may be measured by means of the micrometer, and the cross wires of A must be removed one third of this distance from their first position towards the side opposite to that on which the division appears in the microscope. The line of reference must then be brought again to the wires of the fixed microscope, and the whole operation repeated as before, till the intersections of the wires of A and B, are at the proper distance from each other.

In order to cut the divisions; having ascertained that the line of reference, the division which is under B, and the adjustable dot under A, are accurately in the intersection of the wires of their respective microscopes; turn the circle till A and B, have each an adjustable dot in the intersection of their cross wires, and cut a division; move the circle till an adjustable dot is in the intersection of the wires of B, and a division in that of A, and trace another line with the cutting point. It is now evident, that one of the primary divisions of seventy-two degrees is trisected. Take off the adjustable dots, which are now no longer necessary, and bringing each of the new divisions, as well as the primary ones, successively under the intersection of the wires of the microscope A, trace a new

division with the cutting point, and proceed in this manner round the whole circle, remembering that when a division is in the intersection of the wires of A, another division should be found precisely in that of B, and this affords a very severe test of the accuracy of the work as it proceeds.

Having thus obtained divisions of *twenty-four degrees*, these are again to be trisected precisely in the same manner, and with all the precautions before detailed, placing, should it be necessary, the microscope B, at the distance of two divisions, or forty-eight degrees, from the fixed microscope, and transferring the adjustable dots through two divisions instead of one, which will bring the new divisions under the microscope A, when the adjustable dots may be taken off.

The circle is now divided into portions of *eight degrees* each, and these are to be continually bisected, till we arrive at divisions of *half a degree*; but as it is presumed that the centres of the microscopes A and B, cannot be brought nearer to each other than eight degrees, it becomes necessary to take distant divisions, and to bisect twenty-four degrees. For this purpose, having brought the line of reference to the intersection of the wires of the fixed microscope, and verified the adjustment of the cutting point, place the microscope A, as near to the fixed microscope as convenient, with the intersection of its cross wires half way by estimation between any two divisions, and at the distance of twelve degrees place the microscope B, causing the angles of its cross wires, by means of the micrometer screw, to be bisected by the division under it. Fix an adjustable dot beneath the intersection of the wires of A, and see that it passes through that of B, as formerly directed. Adjust for distinct vision, &c. in the manner which has already

been minutely detailed, and on bringing the adjustable dot to the intersection of the wires of B, if the distance of the microscopes be accurate, a division will be found precisely in the intersection of the wires of A, but should it not be so, the error in the distance of A and B, must be determined and corrected by means of the micrometer, as before described. The line of reference is then to be brought back to the intersection of the wires of the fixed microscope, and the trial repeated.

When the distance of A and B, is found to be correct; the line of reference being brought to the intersection of the cross wires of the fixed microscope, and a division being in that of the microscope B, it is evident that the intersection of the wires of A must accurately bisect a division, or be four degrees distant from the next division on the instrument. Bring up this division therefore to the intersection of the wires of A, and trace a line with the cutting point; do the like with each division, or every eighth degree successively, and the circle will be divided into parts of *four degrees* each, and by continuing to bisect in the same manner, the division of the circle is to be carried to *thirty minutes*.

The last operation consists in trisecting the divisions of thirty minutes.* Bring the line of reference to the fixed microscope, and examine the adjustment of the cutting frame as before. Place the intersections of the wires of B and A, on any two divisions of the instrument, say ten degrees from each other, and increase their distance by advancing the intersection of the wires of A, by means of its micrometer screw, one third of a division, or *ten minutes*. Place an adjustable dot under

* The method by which this trisection is effected is of very general application, and may be used with equal facility to divide an arc into any other number of parts.

A, adjusting the cross wires, and using all the various precautions which have been before minutely detailed. Bring the dot under B, and place a second dot under A: turn the circle till this second dot arrives at B, when if A and B are accurately at the required distance from each other, it is evident that a division will be seen precisely in the intersection of the wires of A; but if not, the error must be corrected as before described, by means of the micrometer. The line of reference is then to be brought back to the fixed microscope, in which position of the circle the intersection of the wires of A will be accurately one third of a division, or ten minutes in advance. Turn the circle, and *bring every division both of thirty minutes and those newly cut*, successively, as they arrive in the microscope, to the intersection of the wires of A, tracing a new division with the cutting point wherever there does not appear to have been one previously made, and continue this till the whole circle is divided into parts of ten minutes each.

In the example of the process of dividing which I have given, the diameter of the circle was supposed to be two feet, but should the instrument be of a smaller size, the microscopes cannot be brought so near to each other as eight degrees, and it then becomes necessary to choose a different series of numbers for the divisions. For this purpose, I have added a small table containing several series which appear to be most convenient, and from among which the artist may select that which may best suit the size of his instrument, and his own ideas of accuracy.

It may not be useless briefly to recapitulate the precautions which are absolutely requisite to the success of this mode of dividing.

The microscopes must be perfectly free from parallax, and the vertical angles formed by the intersections of their cross wires, accurately bisected by the divisions which pass through them. The intersections of the cross wires of A and B must appear between the circles which limit the divisions, and be equidistant from the centre of the instrument to be divided.

The dot on the cutting frame, if moved through the field of view, must always pass through the intersection of the wires of the fixed microscope; and the motion of the cutting point must be in the diameter of the circle to be divided.

When the line of reference is in the intersection of the wires of the fixed microscope, a division should always appear precisely in the intersection of the wires of the microscope B.

Lastly, previous to cutting each new division, if two divisions do not appear precisely in the intersections of the wires of A and B, it is an immediate proof, (unless the microscopes may have suffered any accidental change of position) of inaccurate execution in the former part of the work.

From what has been said, it is evident that the microscope B might have remained stationary after having been placed over one of the primary divisions of seventy-two degrees, and perhaps in practice this may be preferable. My only reason for directing it to be removed was to obviate the effect of any expansion which might take place in that part of the arc which is between the tracing point and the microscope, by placing it as near as possible to the cutting frame; this, however, will probably be found to be an unnecessary refinement.

This method of dividing is not confined to circles, but may be applied with equal facility and advantage to the division of straight lines and zenith sectors. For the last, it is necessary

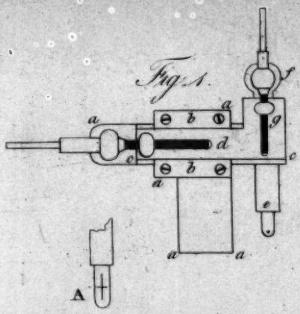


Fig. 1.

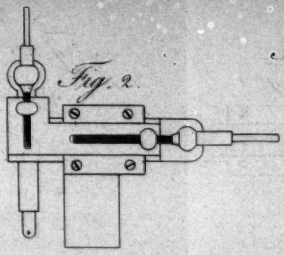


Fig. 2.

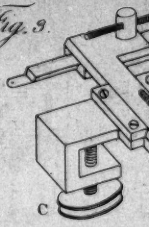


Fig. 3.

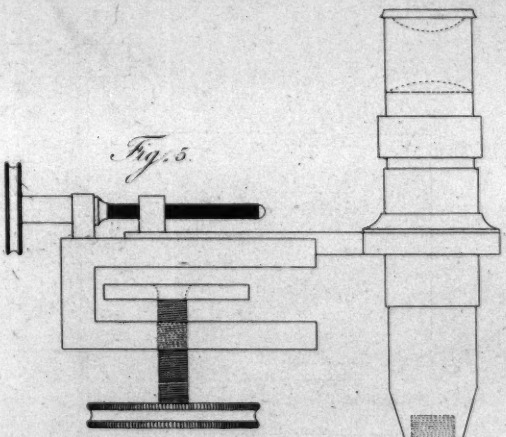


Fig. 5.

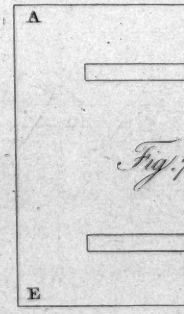


Fig. 7.

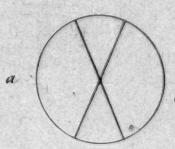


Fig. 6.

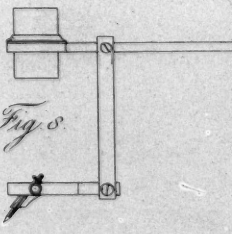
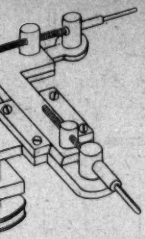


Fig. 8.



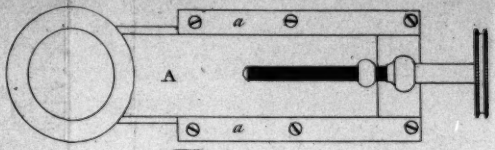
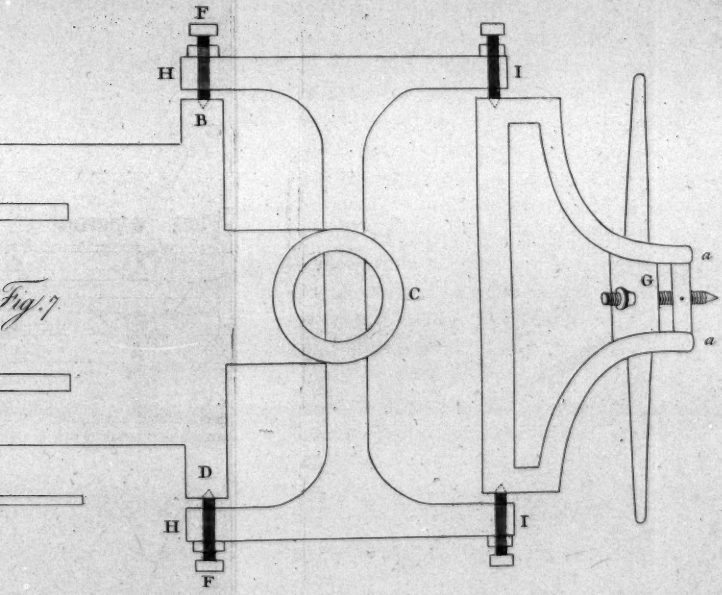


Fig. 4.

Fig. 7.



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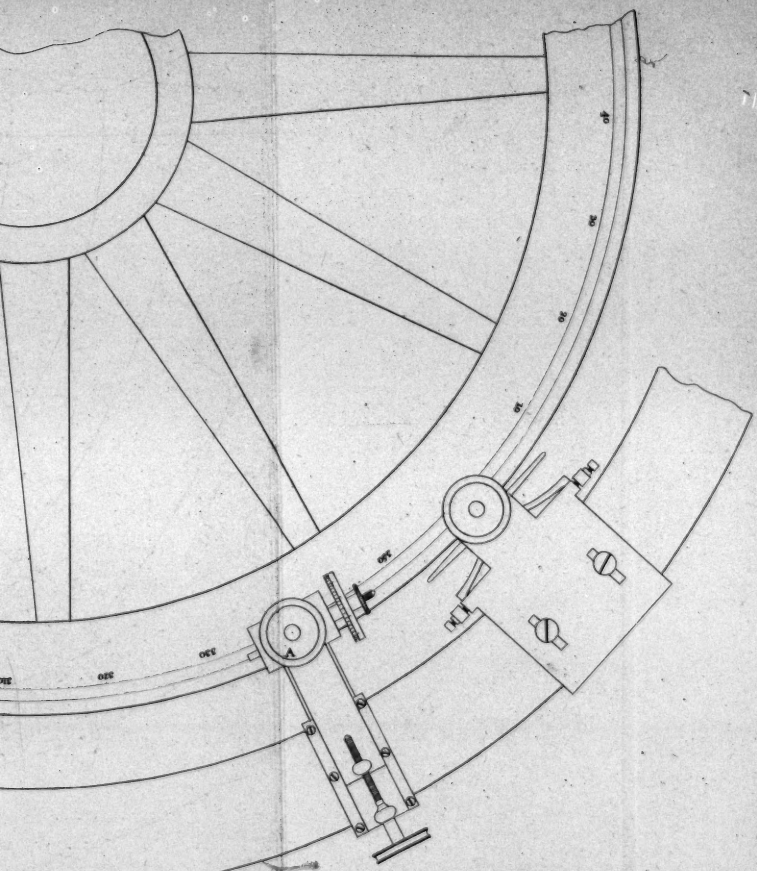
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J. Basire sc.

to obtain the radius of the instrument to be divided, and by continual bisection its eighth part, which is the chord of seven degrees ten minutes. This chord is to be faintly set off on a part of the arc, and the fixed microscope being placed over the middle point of the arc, or zero, the division may be carried to five or ten minutes, precisely in the same manner as has already been directed for the circle.

Finally, whether the operation of dividing is performed by daylight, or by the light of a lamp, I would strongly recommend the use of shades made of thin oiled paper without wire marks, placed very near the limb of the instrument. The accuracy, and, if I may be allowed the expression, the luxury which they afford in delicate observations, can be fully appreciated only by those, who, like myself, have been in the frequent habit of using them.

No. 1. For a circle of two feet and upwards.		No. 2.		No. 3.		No. 4. For a smaller circle.	
$\frac{360}{5}$	will give 72 ° ′ 0	$\frac{360}{2}$	will give 180 ° ′ 0	$\frac{360}{3}$	will give 120 ° ′ 0	$\frac{360}{5}$	will give 72 ° ′ 0
$\frac{72}{3}$	- 24 0	$\frac{180}{2}$	- 90 0	$\frac{120}{3}$	- 40 0	$\frac{72}{3}$	- 24 0
$\frac{24}{3}$	- 8 0	$\frac{90}{2}$	- 45 0	$\frac{40}{5}$	- 8 0	$\frac{24}{2}$	- 12 0
$\frac{24}{2}$	- 4 0	$\frac{45}{3}$	- 15 0	(See No. 1.)		$\frac{36}{2}$	- 6 0
$\frac{20}{2}$	- 2 0	$\frac{30}{3}$	- 5 0			$\frac{42}{3}$	- 2 0
$\frac{18}{2}$	- 1 0	$\frac{15}{2}$	- 2 30			$\frac{26}{2}$	- 1 0
$\frac{10}{2}$	- 0 30	$\frac{32° 30'}{3}$	- 0 50 and 0 10			$\frac{25}{2}$	- 0 30
$\frac{31° 30'}{3}$	- 0 10					$\frac{24° 30'}{2}$	- 0 15

London, January, 1814.

XXI. *Results of some recent Experiments on the Properties impressed upon Light by the Action of Glass raised to different Temperatures, and cooled under different Circumstances. By* David Brewster, *LL. D. F. R. S. Edin. and F. A. S. E. in a Letter to the Right Hon. Sir* Joseph Banks, *Bart. K. B. P. R. S. &c. &c. &c.*

Read May 19, 1814.

Dear Sir,

The interest which you take in every thing which contributes to the progress of science, renders it unnecessary to make any apology for communicating to you the results of some recent experiments on the properties impressed upon light by the action of glass, raised to different temperatures and cooled under different circumstances. The imperfect state of these experiments will still require your indulgence; but I trust that independent of all farther developement, the results themselves will be deemed of sufficient importance to justify me in submitting them so early to your consideration.

During an extensive course of experiments on the depolarisation of light, by soft bodies melted and cooled between plates of glass, it was necessary to bring the plates to different temperatures, in order to fuse the substance which they inclosed. When the body melted at a high temperature and was in a fluid state, I frequently perceived a partial depolarisation of the transmitted light, which gradually went off as the heat diminished. This effect I at first ascribed to an incipient crystallisation in the included substance; but the increase of

the quantity of depolarised light by an augmentation of temperature refuted this conjecture, and rendered it necessary to search for some other cause.

Having procured a plate of glass about $\frac{1}{10}$ of an inch thick, I brought it nearly to the temperature of a red heat. When a pencil of light polarised by reflection was transmitted in a perpendicular direction through the heated glass, the whole of the light was completely depolarised, and found a ready passage through a plate of agate, having its veins perpendicular to the plane of reflection. As the temperature diminished, the quantity of depolarised light seemed to diminish in the same proportion; and after the glass had completely cooled, the whole of the pencil preserved its original polarisation, and refused to penetrate the agate. Hence it follows *that glass brought to a certain temperature forms two images, and polarises them in an opposite manner, like all doubly refracting crystals, the one image being coincident with the other.*

The analogy thus indicated between heated glass and regularly crystallised bodies was in some degree imperfect, as I could perceive no traces of the coloured rings which almost every crystal exhibits by polarised light. It occurred to me, that the temperature of the glass might not have been sufficiently high, and that the phenomenon of the coloured rings could only be developed when the glass was in absolute fusion, or in a state approaching to it. The impracticability, however, of making any experiments of this kind with melted glass, compelled me to have recourse to another expedient.

In the formation of glass tears, or Rupert's drops, as they are sometimes called, by dropping melted glass into cold water, it appeared probable, that in consequence of the sudden conso-

lidation of the outer crust, the interior part of the drop had a structure similar to that of fluid glass, or at least, that the ultimate particles were in both cases at the same distance, having been prevented, in the case of the drop, from approaching each other by the action of the external coat.

I therefore procured several of these drops made of bottle glass, and upon exposing them to a polarised pencil, I found that they not only depolarised it completely, but produced the alternation of the prismatic colours, which I was so anxious to discover. In order to observe this interesting phenomenon with more satisfaction, I obtained several drops made of flint glass. All of them exhibited the same phenomena which appeared in those made of bottled glass, and there was a decided approximation to neutral axes in lines parallel and perpendicular to the axis of the drop. When the polarised ray was transmitted through a part of the drop nearer to the slender stem, the neutral and depolarising axes became still more distinct, till at a certain thickness of the stem these axes were as completely developed as in the most perfect crystals.

Having succeeded in detaching the thick bulb of the drop from its long slender tail, I ground upon it two parallel surfaces, perpendicular to the axis of the drop, and other two parallel to the same axis. When polarised light was transmitted in both these directions, it was depolarised as before, and exhibited the segments of the coloured rings, but there was no appearance of any neutral axes, the depolarisation being equally complete in every position of the drop.

When a piece of plate glass is brought to a red heat, and suddenly cooled by immersion in water, it is intersected by numerous fissures, arranged somewhat like the cells of a

honeycomb; but as the rupture of the surface leaves the particles at liberty to dispose themselves at distances corresponding to the temperature, the glass exhibits no power either in polarising or depolarising light.

The effects of heat upon crystallised bodies, but particularly upon those which, like fluor spar and muriate of soda, do not possess the property either of polarisation or depolarisation, I propose immediately to investigate; but even if the inquiry should be carried no farther, the results which I have described must be considered as of the utmost importance, not only as giving us positive information respecting the structure of crystallised bodies, but as opening a new path to the solution of the great problem of double refraction.

I have the honour to be,

dear Sir,

your most obedient and humble servant,

DAVID BREWSTER.

Edinburgh, April 8th, 1814.

To the Right Hon. Sir Joseph Banks, Bart. K. B. P. R. S.
&c. &c. &c.

XXII. *Consideration of various Points of Analysis.* By John F. W. Herschel, *Esq. F. R. S.*

Read May 19, 1814.

On the Calculus of Generating Functions.

In whatever point of view we consider the theory of generating functions, whether as the fertile source of new discoveries, or as a medium for exhibiting, in the most comprehensive and uniform point of view, results already known; we shall find fresh cause to admire the profound and original genius of its author. To the latter of these objects it is, however, more peculiarly adapted, and perhaps, in the present state of analytics, this may even be considered as the more precious advantage of the two. Such has been the indefatigable activity of those illustrious men, who have devoted themselves to the pursuit of mathematical science, that analysis must be considered as already adequate to every purpose to which we can reasonably hope to see it applied. The attention of the scientific observer must now be directed to those elevated stations, from which distinct and extended views of its arrangement as a whole can be obtained. The calculus of generating functions affords such a station, and commands a wider and more magnificent prospect than any which has yet been opened to the view of the speculative philosopher. It becomes interesting then to extend its application as far as possible in this line, and to introduce it on every occasion where there seems any

probability of its coming successfully into play. Such have been in part the considerations which determined me to adopt it as a vehicle in laying before the Society some results of a singular and interesting nature, derived indeed originally from other principles, but which, like all the rest, flow with the utmost facility from the first elements of this calculus.

In the following pages I have uniformly made use of the functional or characteristic notation; together with the method of separating (where it could conveniently be done) the symbols of operation from those of quantity. This method I have, perhaps, extended and carried somewhat farther than has hitherto been customary; but, I trust, without losing sight of its grand and ultimate object, the union of extreme generality with conciseness of expression. To avoid the necessity of continual explanation, I shall here set down the leading points of the system.

I. The signs : × ( ) are used to separate the symbol of operation from that of the quantity operated upon, thus:

$$f(x), \varphi : \log x, \left\{\frac{d}{dx} - 1\right\}^n \times \varphi(x).$$

II. 1. The combination of two operations is represented by placing their symbols together in their proper order. Thus, $\varphi(\psi(x))$ is simply written $\varphi\psi(x)$. For example, if $\varphi(x) = 1 + x$, and $\psi(x) = x^2$, then $\varphi\psi(x) = 1 + x^2$, and $\psi\varphi(x) = (1 + x)^2$.

2. When several operations combined are considered as one, their characteristics are inclosed in parentheses ( ). Thus

$$f \log \varphi(x) = (f \log \varphi) : x.$$

3. The repetition of the same operation f being denoted (by the first rule of this article) by ff, fff, &c. may be more con-

cisely represented thus, $f^1, f^2, \ldots f^n$; and this furnishes us with a general and very simple notation for the reverse operation of that denoted by f. For since $f^m f^n(x) = f^{m+n}(x)$, if we make $m = -1$, $n = +1$, we find $f^{-1} f(x) = f^0(x) = x$ with the operation f *no times* performed, $= x$. Thus f^{-1} is the characteristic of that operation which must be performed on $f(x)$ to reduce it to x: that is, of the reverse operation. This is surely a simpler and more expressive method than that of inverting the characteristic,* accentuating it on the left side,† or below;‡ or other similar contrivances. For instance,

$$\log^{-1} x = \iota^x = 1 + \frac{x}{1} + \frac{x^2}{1.2} + \&c.$$

$$\tan^{-1} x = \frac{x}{1} - \frac{x^3}{3} + \frac{x^5}{5} - \&c.$$

4. If a combination of operations, as $\phi\psi$ &c. be n times repeated in their order, thus; $\phi\psi\phi\psi \ldots . (x)$, it will, by the second and third rules of this article be denoted by $(\phi\psi)^n : x$. It must be observed, however, that $(\phi\psi)^n$ is not the same as $\phi^n \psi^n$, except in some particular cases.

III. 1. If any number of functions of a symbol x be added, subtracted, or otherwise combined, the resulting function is expressed by the same combination of their characteristics, observing the following conditions.

2. The actual multiplication of two functions $\phi(x)$ and $\psi(x)$ is expressed by inserting a full point between their characteristics, thus, $\phi . \psi(x) = \phi(x) . \psi(x)$.

3. The actual elevation of a function to any power is thus

* LAPLACE. Journal de l'Ecole Polytechnique. No. 15.

† MONGE. Savans Etrangers. 1773.

‡ KNIGHT. Philosophical Transactions, 1811. Part I.

expressed, $\left\{f(x)\right\}^n = \left\{f\right\}^n : x$, to distinguish it from $f^n(x)$, whose signification has already been explained.

4. If F (x) be developable in any series of the form

$$ax^\alpha + bx^\beta + cx^\gamma + \&c.$$

the following abbreviations are used

$$(\mathrm{F} : \varphi) : x = a \cdot \varphi^\alpha(x) + b \cdot \varphi^\beta(x) + c \cdot \varphi^\gamma(x) + \&c.$$

and, $(\mathrm{F}\left\{\varphi\right\}) : x = a \cdot \left\{\varphi(x)\right\}^\alpha + b \cdot \left\{\varphi(x)\right\}^\beta + \&c.$

Thus (for example)

$$\frac{1}{1-\frac{d}{dx}} \times y = y + \frac{dy}{dx} + \frac{d^2y}{dx^2} + \&c.$$

$$\frac{1}{1-\left\{\log.^{-1}\right\}} \times x = x + \varepsilon^x + \varepsilon^{2x} + \&c.$$

IV. 1. D is used as the sign of derivation. It is, properly speaking, the sign of an operation performed, not on quantity, but on the characteristic which it immediately precedes; by which the operation denoted by that characteristic is altered. For instance: D sin. $=$ cos.; D cos. $= -$ sin. But it must be observed that D log. $^{-1} =$ log. $^{-1}$.

2. The sign D affects only the characteristic next following it, thus, $\mathrm{D}\varphi f(x) = (\mathrm{D}\varphi) : f(x)$. If it be intended to affect a combination of operations, the rule II, 2 must be observed. Thus, D $(\varphi f) : x$, D^n $(\psi \log.^{-1}) :$ log. x.

V. Every functional characteristic is affected by all the characteristics preceding it, in the same manner as if it were a symbol of quantity.

VI. Every characteristic of operation *performed on quantity* affects all which follows it, as if it were one symbol. Thus if $f(x) = ax^\alpha + bx^\beta +$ &c.; we shall have

$$f\mathrm{D}\phi(x) = a \cdot \left\{\mathrm{D}\phi(x)\right\}^{\alpha} + b \cdot \left\{\mathrm{D}\phi(x)\right\}^{\beta} + \&c.$$

This rule does not extend to the signs D, d, Δ, δ, f, Σ, according to the remark in IV. 1.

These rules will suffice to explain whatever may appear obscure or capricious in the following sheets. We shall now proceed to their practical application.

If $\phi(t)$ be a function of t, developable in a series

$$\mathrm{A}_{-\infty}t^{-\infty} + \ldots\ldots \mathrm{A}_0 + \mathrm{A}_1 t + \ldots\ldots \mathrm{A}_x t^x + \ldots \mathrm{A}_{\infty} t^{\infty}$$

$\phi(t)$ is said to be the generating function of A_x, and it may be said to be taken with respect to t. To this we shall appropriate a peculiar symbol G_t, as follows:

$$\phi(t) = \mathrm{G}_t \left\{\mathrm{A}_x\right\}$$

When only one symbol t is used, the index below the G may be understood, and our equation will be

$$\phi(t) = \mathrm{G}\left\{\mathrm{A}_x\right\}.$$

If $\phi(t, t')$ be a function of t, t', developable in a double series

$$\begin{aligned}
&\mathrm{A}_{-\infty,-\infty}\, t^{-\infty}.t'^{-\infty} + \ldots\ldots \mathrm{A}_{-\infty,y}\, t^{-\infty}.t'^{y} + \ldots \mathrm{A}_{-\infty,\infty}\, t^{-\infty}.t'^{\infty} \\
&+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&+ \mathrm{A}_{x,-\infty}\, t^{x}.t'^{-\infty} + \ldots\ldots \mathrm{A}_{x,y}\, t^{x}.t'^{y} + \ldots\ldots \mathrm{A}_{x,\infty}\, t^{x}.t'^{\infty} \\
&+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&+ \mathrm{A}_{\infty,-\infty}\, t^{\infty}.t'^{-\infty} + \ldots\ldots \mathrm{A}_{\infty,y}\, t^{\infty}.t'^{y} + \ldots\ldots \mathrm{A}_{\infty,\infty}\, t^{\infty}.t'^{\infty}.
\end{aligned}$$

$\phi(t, t')$ is said to be the generating function of $\mathrm{A}_{x,y}$ with respect to t, t', and may be thus expressed;

$$\phi(t, t') = \mathrm{G}'_{t,t'}\left\{\mathrm{A}_{x,y}\right\}$$

and so on, if there be any number of symbols t, t', t'', &c. To

denote the sum of all the terms of a series, we shall use the sign S, thus,

$\phi(t) = S\left\{A_x t^x\right\}$, and in like manner $\phi(t, t') = S_{x,y}\left\{A_{x,y} \cdot t^x \cdot t'^y\right\}$.

For t let ht be written, and we obtain

$$\phi(ht) = S\left\{A_x (ht)^x\right\} = S\left\{A_x h^x \cdot t^x\right\} = G\left\{A_x h^x\right\}$$

Thus, if the generating function of A_x be $\phi(t)$, that of $A_x h^x$ will be $\phi(ht)$.

Let this equation be multiplied by t^{-r}, and we get

$$t^{-r}\phi(ht) = S\left\{A_x h^x \cdot t^{x-r}\right\}$$
$$= S\left\{A_{x+r} h^{x+r} \cdot t^x\right\} = G\left\{A_{x+r} \cdot h^{x+r}\right\}.$$

If then the generating function of A_x be $\phi(t)$, that of $A_{x+r} h^{x+r}$ will be $t^{-r} \cdot \phi(ht)$.

Again, it is easy to see that

$$a \cdot G\left\{A_x\right\} + b \cdot G\left\{B_x\right\} + \&c. = G\left\{aA_x + bB_x + \&c.\right\}$$

and thus we have

$$(at^{-\alpha} + bt^{-\beta} + ct^{-\gamma} + \&c.) \cdot \phi(ht) = G\left\{aA_{x+\alpha} h^{x+\alpha} + bA_{x+\beta} h^{x+\beta} + \&c.\right\}$$

and, if $h = 1$

$$(at^{-\alpha} + bt^{-\beta} + \&c.) \cdot \phi(t) = G\left\{aA_{x+\alpha} + bA_{x+\beta} + \&c\right\}; (1).$$

Let us express the function $aA_{x+\alpha} + bA_{x+\beta} + \&c$ by the symbol ∇A_x and let

$$at^\alpha + bt^\beta + ct^\gamma + \&c. = f(t)$$

∇A_x then will be the same as $f(A_x)$, provided that in the

development of $f(A_x)$, A_{x+i} be every where written for $\{A_x\}^i$, and to derive the generating function of ∇A_x from that of A_x, we have only to multiply the latter by $f\left(\frac{1}{t}\right)$, thus

$$G\left\{\nabla A_x\right\} = \varphi(t) \cdot f\left\{\frac{1}{t}\right\}; \quad . \quad . \quad . \quad (2)$$

Let now $\nabla^1 A_x$, $\nabla^2 A_x$, &c. denote the respective values of the expression

$$aA_{x+\alpha} + bA_{x+\beta} + \&c.$$

When instead of A_x we write ∇A_x, $\nabla^1 A_x$, &c. that is,

$$\nabla^{i+1} A_x = a\nabla^i A_{x+\alpha} + b\nabla^i A_{x+\beta} + \&c.$$

and it is easy to see that we shall have

$$G\left\{\nabla^i A_x\right\} = \left\{f\left(\frac{1}{t}\right)\right\}^i \cdot \varphi(t); \quad . \quad . \quad . \quad . \quad (3).$$

Let us now denote by $'f\left(\frac{1}{t}\right)$ the expression

$$'a \cdot t^{-'\alpha} + 'b \cdot t^{-'\beta} + \&c.$$

and by $'\nabla^i A_x$ the function formed from $'f\left(\frac{1}{t}\right)$ in the same manner as we formed $\nabla^i A_x$ from $f\left(\frac{1}{t}\right)$. The equation (3) then may be thus written

$$G\left\{\nabla^i A_x\right\} = \left\{(ff^{-1}) : 'f\left(\frac{1}{t}\right)\right\}^i \cdot \varphi(t)$$

If then $\left\{f'f^{-1}(t)\right\}^i$ be developed by the ordinary methods into a series of the form

$$a_{-\infty} t^{-\infty} + \ldots\ldots a_x t^x + \ldots\ldots a_\infty t^\infty = S_x\left\{a_x t^x\right\}$$

we shall obtain

$$G\left\{\nabla^i A_x\right\} = S_x\left\{a_x \cdot \left\{'f\left(\frac{1}{t}\right)\right\}^x\right\} \cdot \varphi(t) = S_x\left\{a_x \cdot G\left\{'\nabla^x A_x\right\}\right\},$$

and of course

$$\nabla^i A_x = a_{-\infty}\,{}'\nabla^{-\infty} A_x + \ldots\ldots a_0 A_x + a_1\,{}'\nabla A_x + \ldots\ldots$$
$$a_\infty\,{}'\nabla^\infty A_x; \ldots\ldots\ldots (4).$$

Thus we may always develope $\nabla^i A_x$ in a series containing only the successive orders of ${}'\nabla A_x$, such as ${}'\nabla^2 A_x$, &c.

If the developement of $\left\{f'f^{-1}(t)\right\}^i$ contain no negative powers of t, we have

$$a_x = \frac{D^x \left\{ff^{-1}\right\}^i : o}{1.2 \ldots\ldots x}$$

and consequently

$$\nabla^i A_x = \left\{ff^{-1}\right\}^i : o \,.\, A_x + \frac{D\left\{ff^{-1}\right\}^i : o}{1} \,.\, {}'\nabla A_x + \frac{D^2\left\{ff^{-1}\right\}^i : o}{1.2}$$
$${}'\nabla^2 A_x + \&c. \ldots\ldots\ldots (5).$$

Let ${}'\nabla A_x = A_{x+1} - A_x$, and we have $f\left(\frac{1}{t}\right) = \frac{1}{t} - 1$, and ${}'f^{-1}(t) = 1 + t$, whence we obtain

$$\nabla^i A_x = \left\{f(1)\right\}^i \,.\, A_x + \frac{D\left\{f\right\}^i : 1}{1} \Delta A_x + \frac{D^2\left\{f\right\}^i : 1}{1.2} \,.\, \Delta^2$$
$$A_x + \&c.; \ldots\ldots\ldots\ldots (6)$$

for it is evident that when $t = o$, $D^x\left\{f(1+t)\right\}^i$ becomes $D^x\left\{f\right\}^i : 1$. To take a particular case, let $\nabla A_x = A_{x+1}$ and $\nabla^i A_x = A_{x+i}$, $f\left(\frac{1}{t}\right) = \frac{1}{t}$, and $D^x\left\{f\right\}^i : 1 = i(i-1)\ldots\ldots$ $(i - x + 1)$, whence

$$A_{x+i} = A_x + \frac{i}{1} \,.\, \Delta A_x + \frac{i(i-1)}{1.2} \,.\, \Delta^2 A_x + \&c. \quad (7).$$

Again, if we suppose $\nabla A_x = \Delta A_x = A_{x+1} - A_x$, and ${}'\nabla A_x = A_{x+1}$, we shall obtain from (5)

$$\Delta^i A_x = A_{x+i} - \frac{i}{1} \cdot A_{x+i-1} + \frac{i(i-1)}{1.2} \cdot A_{x+i-2} - \&c; \ldots (8)$$

But to proceed. We have,

$$G\left\{x^i A_x\right\} = S\left\{A_x x^i \cdot t^x\right\} = \frac{t}{dt} d \frac{t}{dt} d \ldots\ldots \frac{t}{dt} \cdot d\phi(t)$$

or which is the same thing,

$$G\left\{x^i A_x\right\} = \frac{1}{d \log t} d \frac{1}{d \log t} d \ldots\ldots d\left\{\phi \log^{-1} : \log t\right\}$$

Now,

$$\frac{d\left\{\phi \log^{-1} : \log t\right\}}{d \log t} = \frac{D(\phi \log^{-1}) : \log t \times d \log t}{d \log t} = (\phi \log^{-1}) : \log t$$

$$\frac{d\left\{D(\phi \log^{-1}) : \log t\right\}}{d \log t} = D^2(\phi \log^{-1}) : \log t$$

and so on. Thus our equation becomes

$$G\left\{x^i A_x \cdot\right\} = D^i(\phi \log^{-1}) : \log t$$

and, if $f(x) = ax^\alpha + bx^\beta + \&c$, we see that

$$G\left\{A_x f(x)\right\} = (aD^\alpha + bD^\beta + \&c)(\phi \log^{-1}) : \log t$$
$$= (f : D)(\phi \log^{-1}) : \log t ; \ldots\ldots (9)$$

If $f(x)$ be a rational integral function of x, the second member of this equation will require only the ordinary rules of the differential calculus for its formation, and of course the first may be rigorously obtained.

Conceive $f(t)$ and $F(t')$ to be developed into the two series $S_x\left\{a_x t^x\right\}$ and $S_y\left\{A_y t^y\right\}$, and let us consider the double series

$$\sigma = S^2_{x,y}\left\{a_x A_y \cdot h^{xy} \cdot t^x \cdot t'^y\right\}$$

First, σ may be expressed as follows,

$$\sigma = S_x\left\{a_x t^x \cdot S_y\left\{A_y \cdot (h^x t')^y\right\}\right\} = S_x\left\{a_x t^x \cdot F(t' h^x)\right\}$$

But by a similar mode of reasoning, we should also find

$$\sigma = S_y \left\{ A_y t^{y} . f(th^{y}) \right\}$$

or, $$\sigma = S_x \left\{ A_x t^{x} . f(th^{x}) \right\}$$

for since x and y vary through all their values, these two sums are identical. Equating then the values of σ

$$S \left\{ A_x t^{x} . f(th^{x}) \right\} = G_t \left\{ a_x . F(t^{\prime} h^{x}) \right\}$$

$$= (F : t^{\prime} h^{D}) (f \log^{-1}) : \log t$$

Let $t^{\prime} = 1$, and for h writing h^{-1}, and adding or subtracting

$$S\left\{ A_x . \frac{f(tb^{x}) \pm f(tb^{-x})}{2} \right\} = \frac{F(b^{D}) \pm F(b^{-D})}{2} (f \log^{-1}) : \log t; \quad (10)$$

II. *On Logarithmic Transcendents.*

The equation we have just arrived at affords us a method of exhibiting, in a finite form, the sum of the series in its first member, provided we possess the means of obtaining the second; and it appears, by what we have before remarked, that this can be performed, whenever $F(h^{D}) \pm F(h^{-D})$ is a rational integral function of D. This includes among the forms of F those remarkable functions denominated by Mr. SPENCE, "Logarithmic Transcendents," and we shall now proceed, by the help of the general property demonstrated by that author in his "Essay &c." to derive from these principles the summation of one of the most extensive classes of series which has yet received discussion.

Adopting Mr. SPENCE's notation, we will represent the series

$$\frac{x}{1^{n}} - \frac{x^{2}}{2^{n}} + \frac{x^{3}}{3^{n}} - \&c.$$

by the symbol ${}^{n}L(1 + x)$. The property then alluded to is as follows:

$$\frac{{}^{n}\mathrm{L}(1+x)+(-1)^{n}\cdot{}^{n}\mathrm{L}(1+x^{-1})}{2}=\frac{{}^{1}\mathrm{L}(2)\cdot(\log.x)^{n}}{1.2\ldots n}+\frac{{}^{3}\mathrm{L}(2)\cdot(\log.x)^{n-2}}{1.2\ldots(n-2)}+\&c.$$

continued as far as it will go without involving negative powers of log. x. Supposing then $\mathrm{F}(t)={}^{n}\mathrm{L}(1+t)$, and writing ϵ^{θ} for h we obtain

$$\mathrm{S}\left\{\frac{(-1)^{x+1}}{x^{n}}\cdot\frac{f(\epsilon^{x\theta})+(-1)^{n}\cdot f(\epsilon^{-x\theta})}{2}\right\}=$$

$$=\left(\frac{{}^{1}\mathrm{L}(2)\cdot\theta^{n}}{1.2\ldots n}\mathrm{D}^{n}+\frac{{}^{3}\mathrm{L}(2)\cdot\theta^{n-2}}{1.2\ldots(n-2)}\mathrm{D}^{n-2}+\&c.\right)(f\log^{-1}):\log t;\quad(11)$$

A very remarkable case of this equation is when $t=1$, or $\log t=0$, for then $\frac{\mathrm{D}^{i}(f\log^{-1}):\log t}{1.2\ldots i}=$ the coefficient of t^{i} in the developement of $f\log^{-1}(t)$ or of $f(\epsilon^{t})$. If then we suppose

$$f(\epsilon^{t})=a_{0}+a_{1}t+a_{2}t^{2}+\ldots a_{n}t^{n}+\&c.$$

we shall have the following equation:

$$\mathrm{S}\left\{\frac{(-1)^{x+1}}{x^{n}}\cdot\frac{f(\epsilon^{x\theta})+(-1)^{n}\cdot f(\epsilon^{-x\theta})}{2}\right\}={}^{0}\mathrm{L}(2)\cdot a_{n}\theta^{n}+{}^{2}\mathrm{L}(2)\cdot a_{n-2}\theta^{n-2}+\&c.;\quad(12)$$

The second member being continued so long as it does not involve negative powers of θ.

With regard to the functions ${}^{1}\mathrm{L}(2)$, ${}^{3}\mathrm{L}(2)$, &c. we have, as is well known

$${}^{1}\mathrm{L}(2)=1-1+1-1+\&c.=\tfrac{1}{2},\text{ and}$$

$${}^{2x}\mathrm{L}(2)=\frac{(2^{2x-1}-1)\,\pi^{2x}}{1.2\ldots\ldots(2x)}\cdot\mathrm{B}_{2x-1}$$

B_{2x-1} being the xth number of BERNOUILLI.

The equation (12) by assigning specific forms to $f(t)$ affords an indefinite variety of interesting results, of which we shall only notice a few, the most remarkable.

1. Let $f(t)=\frac{-1}{\sqrt{-1+t}}$. and for n write $2n-1$, and $\theta\sqrt{-1}$ for θ.

The usual exponential expression for tan. θ, reduced into two fractions, gives

$$\tan.\,\theta = \frac{-1}{\sqrt{-1}+\epsilon^{\theta\sqrt{-1}}} - \frac{-1}{\sqrt{-1}+\epsilon^{-\theta\sqrt{-1}}}; \quad \ldots\ldots\ldots\ldots (13)$$

Thus, the first member of (12) becomes

$$\tfrac{1}{2}\,S\left\{(-1)^{x+1}\cdot\frac{\tan.\,x\theta}{x^{2n-1}}\right\} \text{ or, } \tfrac{1}{2}\left\{\frac{\tan.\,\theta}{1^{2n-1}} - \frac{\tan.\,2\theta}{2^{2n-1}} + \frac{\tan.\,3\theta}{3^{2n-1}} - \&c.\right\}$$

In order to obtain the coefficients a_1, a_3, $\ldots$ a_{2n-1}, in the second, we have,

$$f(\epsilon^t) = \frac{-1}{\sqrt{-1}+\epsilon^t} = a_0 + a_1\cdot t + a_2 t^2 + \&c.$$

Now $f(\epsilon^t)$ may also be thrown into the form

$$\tfrac{1}{2}\left\{\frac{-1}{\sqrt{-1}+\epsilon^t} - \frac{-1}{\sqrt{-1}+\epsilon^{-t}}\right\} + \tfrac{1}{2}\left\{\frac{-1}{\sqrt{-1}+\epsilon^t} + \frac{-1}{\sqrt{-1}+\epsilon^{-t}}\right\}$$

which is the same as

$$\tfrac{1}{2}\tan.\left(\frac{t}{\sqrt{-1}}\right) + \tfrac{1}{2}\sqrt{-1} - \tfrac{1}{2}\sec.\left(\frac{t}{\sqrt{-1}}\right)$$

Thus the even values of a_x are given by the developement of sec. $\left(\frac{t}{\sqrt{-1}}\right)$ and the odd by that of tan. $\left(\frac{t}{\sqrt{-1}}\right)$ and hence it is easy to see that

$$a_{2x-1} = \frac{(-1)^{x+1}}{\sqrt{-1}}\cdot\frac{2^{2x-1}\cdot(2^{2x}-1)}{1.2.3.\ldots.(2x)}\cdot B_{2x-1}; \qquad (14)$$

and

$$a_{2x} = \frac{-1}{1.2\ldots(2x).2^{2x+1}}\left\{1^{2x} - \left\{3^{2x} - \frac{2x+1}{1}\cdot 1^{2x}\right\} + \left\{5^{2x}\right.\right.$$
$$\left.\left. - \&c.\right\} - \&c.\right\}; \quad (15)$$

but a general value of a_x may readily be obtained by the immediate consideration of the function $f(\epsilon^t)$ itself, as follows:

We know that

$$a_x = \frac{1}{1.2....x} \cdot \mathrm{D}^x \frac{-1}{\sqrt{-1} + \left\{\log^{-1}\right\}} : 0$$

Now we have

$$\mathrm{D}(f \log^{-1}) : t = \frac{-\epsilon^t}{(\sqrt{-1} + \epsilon^t)^2}$$

$$\mathrm{D}^2 (f \log^{-1}) : t = \frac{\epsilon^{2t} - \sqrt{-1}.\epsilon^t}{(\sqrt{-1} + \epsilon^t)^3}$$

. . . . . .

$$\mathrm{D}^x (f \log^{-1}) : t = \frac{{}^0\mathrm{A}.\epsilon^{xt} + {}^1\mathrm{A}.\epsilon^{(x-1)t} + \ldots {}^{x-1}\mathrm{A}.\epsilon^t}{(\sqrt{-1} + \epsilon^t)^{x+1}}$$

In order to determine the numerator of this fraction, we shall adopt the elegant artifice used by LAPLACE * on a similar occasion.

$${}^0\mathrm{A}.\epsilon^{xt} + \ldots {}^{x-1}\mathrm{A}.\epsilon^t = (\sqrt{-1} + \epsilon^t)^{x+1} . \mathrm{D}^x \left\{\frac{-1}{\epsilon^t + \sqrt{-1}}\right\}$$

$$= -(\sqrt{-1} + \epsilon^t)^{x+1} . \mathrm{D}^x \left\{\epsilon^{-t} - \sqrt{-1}.\epsilon^{-2t} - \epsilon^{-3t} + \sqrt{-1}. \&c.\right\}$$

$$= (-1)^{x+1} . (\sqrt{-1} + \epsilon^t)^{x+1} . \left\{1^x.\epsilon^{-t} - \sqrt{-1}.2^x.\epsilon^{-2t} - 3^x.\epsilon^{-3t} + \&c.\right\}$$

Now, as this equation is rigorous, and the first member contains only positive powers of ϵ^t, the negative powers in the second must destroy each other, and may therefore be neglected. Expanding then $(\epsilon^t + \sqrt{-1})^{x+1}$ in powers of ϵ^t, multiplying together the two series, and retaining only positive powers of ϵ^t, we find

* LAPLACE. Mém. de l'Acad. 1779. Sur l'usage du calc. des diff. partielles dans la théorie des suites.

$$^{\circ}A \cdot \varepsilon^{xt} + \ldots {}^{x-1}A \cdot \varepsilon^{t} =$$

$$= (-1)^{x+1} \cdot \left\{ \begin{array}{l} 1^x \cdot \varepsilon^{xt} - \left\{ 2^x - \frac{x+1}{1} \cdot 1^x \right\} \sqrt{-1} \cdot \varepsilon^{(x-1)t} \\ - \left\{ 3^x - \frac{x+1}{1} \cdot 2^x + \frac{(x+1) \cdot x}{1.2} \cdot 1^x \right\} \cdot \varepsilon^{(x-2)t} + \&c. \end{array} \right\}$$

And after the substitution of 0 for t, or 1 for ε^t and its powers we obtain,

$$a_x = \frac{(-1)^{x+1}}{1.2 \ldots x(1+\sqrt{-1})^{x+1}} \times$$

$$\left\{ \begin{array}{l} 1^x - \left\{ 2^x - \frac{x+1}{1} 1^x \right\} \sqrt{-1} \\ - \left\{ 3^x - \frac{x+1}{1} \cdot 2^x + \frac{(x+1) \cdot x}{1.2} \cdot 1^x \right\} + \&c. \end{array} \right\}; \ldots . (16)$$

The equation (12) will thus take the following form,

$$S\left\{(-1)^{x+1} \cdot \frac{\tan x\theta}{x^{2n-1}}\right\} = Y_1 \theta + Y_3 \theta^3 + \ldots Y_{2n-1} \theta^{2n-1}; (17)$$

where

$$Y_{2x-1} = \frac{2^{2x} (2^{2x}-1) \cdot (2^{2n-2x-1}-1) \cdot \pi^{2n-2x}}{1.2 \ldots (2x) \times 1.2 \ldots (2n-2x)} \cdot B_{x-1} \cdot B_{n-x-1}; (18)$$

2. Retaining the same form of the function f, and of course the same value of a_x, for n write $2n$, and for θ, $\theta \cdot \sqrt{-1}$, and the first member of (12) becomes

$$\tfrac{1}{2}\left\{ \sqrt{-1} \cdot {}^{2n}L(2) - S\left\{ \frac{(-1)^{x+1}}{x^{2n} \cdot \cos x\theta} \right\} \right\}$$

And the second,

$$a_0 \cdot {}^{2n}L(2) - a_2 \cdot {}^{2n-2}L(2) \cdot \theta^2 + \ldots + (-1)^n \cdot {}^{0}L(2) \cdot \theta^{2n} \cdot a_{2n}$$

Now, since $a_0 = \frac{-1}{1+\sqrt{-1}}$, collecting into one the coefficients of ${}^{2n}L(2)$, viz.: $\sqrt{-1} - 2a_0$, we find their sum equal to 1, and of course,

$$S\left\{\frac{(-1)^{x+1}}{x^{2n}\cdot\cos x\theta}\right\} = {}^{2n}L(2) + Y_2\theta^2 + Y_4\theta^4 + \ldots Y_{2n}\theta^{2n}; \quad (19)$$

where

$$Y_{2x} = \frac{(-1)^x(2^{2n-2x-1}-1)\cdot\pi^{2n-2x}}{1.2\ldots(2x)\times 1.2\ldots(2n-2x)\cdot 2^{2x}} B_{2n-2x-1}\cdot$$

$$\cdot\left\{\begin{array}{l} 1^{2x} - \left\{3^{2x} - \frac{2x+1}{1}\cdot 1^{2x}\right\} \\ + \left\{5^{2x} - \&c\right\} - \&c \end{array}\right\}; \quad (20)$$

If in this equation for θ we write $\pi + \theta$, we shall obtain the sum of the series

$$\frac{1}{1^{2n}\cdot\cos\theta} + \frac{1}{2^{2n}\cdot\cos 2\theta} + \frac{1}{3^{2n}\cdot\cos 3\theta} + \&c; \ldots\ldots (21)$$

And by addition, of

$$\frac{1}{1^{2n}\cdot\cos\theta} + \frac{1}{3^{2n}\cdot\cos 3\theta} + \frac{1}{5^{2n}\cdot\cos 5\theta} + \&c; \ldots\ldots (22)$$

3. Let $f(t) = \frac{\sqrt{-1}-t}{\sqrt{-1}+t}$, and let $2n$ be written for n and $\theta\sqrt{-1}$ for θ; and the first member of (12) becomes

$$\sqrt{-1}\cdot S\left\{\frac{(-1)^{x+1}}{x^{2n}\cdot\cos x\theta}\right\}$$

Now the developement of $\frac{-1}{\sqrt{-1}+t}$ being $a_0 + a_1 t + \&c$, that of $f(\epsilon^t)$ will be

$$(\epsilon^t - \sqrt{-1})\left\{a_0 + a_1 t + \&c\right\} =$$

$$= \left\{(1-\sqrt{-1}) + \frac{t}{1} + \frac{t^2}{1.2} + \&c\right\}\cdot\left\{a_0 + a_1 t + \&c\right\}$$

and the coefficient of t^x will be found equal to

$$(1-\sqrt{-1})\cdot a_x + \frac{a_{x-1}}{1} + \frac{a_{x-2}}{1.2} + \ldots\ldots \frac{a_0}{1.2\ldots x}$$

Thus the application of (12) gives the following equation

$$S\left\{\frac{(-1)^{x+1}}{x^{2n}.\cos x\theta}\right\} = {}^{2n}L(2) + Y_2\theta^2 + Y_4\theta^4 + \ldots Y_{2n}\theta^{2n}$$

where

$$Y_{2x} = \frac{(-1)^x.(2^{2n-2x-1}-1)\pi^{2n-2x}}{\sqrt{-1}.1.2.....(2n-2x)} B_{2n-2x-1}\left\{(1-\sqrt{-1})\right.$$

$$\left. a_{2x} + \frac{a_{2x-1}}{1} + \ldots \frac{a_0}{1.2...(2x)}\right\}$$

Which compared with the value of Y_{2x} before found (20) gives the following singular equations,

$$0 = a_{2x} + \frac{a_{2x-2}}{1.2} + \frac{a_{2x-4}}{1.2.3.4} + \ldots \frac{a_0 - \frac{1}{2}\sqrt{-1}}{1.2...(2x)}; \ldots\ldots (23)$$

$$a_{2x} = \sqrt{-1}.\left\{\frac{a_{2x-1}}{1} + \frac{a_{2x-3}}{1.2.3} + \ldots \frac{a_1}{1.2...(2x-1)}\right\} - \frac{1}{2}.\frac{1}{1.2...(2x)}. \quad (24)$$

The latter of these two equations affords the even values of a_x in terms of the odd, and hence we are enabled to express the sum of the series $\frac{1}{1^{2x+1}} - \frac{1}{3^{2x+1}} + \frac{1}{5^{2x+1}} - \&c = {}^{2x+1}C(1)$* by means of the numbers of BERNOUILLI, which EULER appears to have considered as impracticable.† We need hardly re-

* See "Essay on Logarithmic Transcendents," page 51. I should not omit to observe that the equations in p. 69 of that work, expressing the value of the function ${}^nC(x) + (-1)^n.{}^nC(x^{-1})$ when combined with our equation (10) by making $F(t) = {}^nC(t) = \frac{t}{1^n} - \frac{t^3}{3^n} + \&c$, afford a series of results, highly interesting, but which the necessary limits of this paper forbid me at present to dilate on.

† "Per hos autem numeros Bernouillianos secans *exprimi non potest, sed requirit* "*alios numeros* qui in summas potestatum reciprocarum imparium ingrediuntur, si "enim ponatur,

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \&c = \alpha.\frac{\pi}{2^2}$$

$$1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \&c = \frac{\beta}{1.2}.\frac{\pi^3}{2^3}, \&c.$$

mark that the imaginary form here assumed by a_{2x} is merely apparent.

To proceed. Let the equation (17), multiplied by $d\theta$, be integrated between the limits o and θ, and we find after all reductions.

$$(\sec\theta)^{\left(\frac{1}{1}\right)^{2n}} \cdot (\cos 2\theta)^{\left(\frac{1}{2}\right)^{2n}} \cdot (\sec 3\theta)^{\left(\frac{1}{3}\right)^{2n}} \cdot \&c. = \log^{-1}\left\{o + Y_1 \cdot \frac{\theta^2}{2} + \ldots\ldots Y_{2n-1} \cdot \frac{\theta^{2n}}{2n}\right\}; \ldots (25).$$

The value of Y_{2x-1} being given by the equation (18)

Again, if we suppose, for the sake of brevity

$$^{2n}L(2) + Y_2 \cdot \theta^2 + \ldots Y_{2n}\theta^{2n} = U(\theta)$$

and $^{2n}L(2) \cdot \frac{\theta}{1} + Y_2 \cdot \frac{\theta^3}{3} + \ldots Y_{2n} \cdot \frac{\theta^{2n+1}}{2n+1} = D^{-1}U(\theta)$, we shall obtain, by operating in the same manner on (19) and the equations derived from it, expressing the values of the series (21) and (22).*

"erit $\alpha = 1, \beta = 1, \gamma = 5$, &c. ex hisque valoribus obtinebitur

$$\sec x = \alpha + \frac{\beta}{1.2}x^2 + \frac{\gamma}{1.2.3.4}x^4 + \&c."$$

EULER. Inst. Calc. Diff. Pars posterior. Cap. VIII. p. 542.

The general value of $^{2x+1}C(1)$ as deduced from our equation (24) in terms of the numbers of BERNOUILLI is as follows;

$$^{2x+1}C(1) = \left(\frac{\pi}{2}\right)^{2x+1} \cdot \left\{\frac{2^{2x-1}(2^{2x}-1)}{1.2.3\ldots(2x)} \cdot \frac{B_{2x-1}}{1} - \frac{2^{2x-3}(2^{2x-2}-1)}{1.2\ldots(2x-2)} \cdot \frac{B_{2x-3}}{1.2.3} + \ldots\ldots\ldots\ldots + (-1)^{x-1} \cdot \frac{2 \cdot (2^2-1)}{1.2} \cdot \frac{B_1}{1.2\ldots(2x-1)} + (-1)^x \cdot \frac{1}{2} \cdot \frac{1}{1.2\ldots(2x)}\right\}.$$

* The constant added to complete these integrals is determined by making $\theta = o$ in which case since $\cot\left(\frac{\pi}{4} - x\theta\right) = 1$, their first members vanish when n is greater

$$S\left\{\frac{(-1)^{x+1}}{x^{2n+1}}\log\cot\left(\frac{\pi}{4}-\frac{x\theta}{2}\right)\right\}=D^{-1}U(\theta);\ldots\ldots(26)$$

$$S\left\{\frac{1}{x^{2n+1}}.\log\cot\left(\frac{\pi}{4}-\frac{x\theta}{2}\right)\right\}=D^{-1}U(\pi)-D^{-1}U(\pi+\theta);(27)$$

$$S\left\{\frac{1}{(2x-1)^{2n+1}}\log\cot\left(\frac{\pi}{4}-\frac{(2x-1)\theta}{2}\right)\right\}=$$

$$\frac{D^{-1}\times\left\{U(\pi)+U(\theta)-U(\pi+\theta)\right\}}{2};\ldots\ldots\ldots\ldots(28).$$

In the last of these equations, if we write $-\left(\frac{\pi}{2}+2\theta\right)$ for θ, we obtain the following equation, corresponding to (25)

$$(\tan\theta)^{\left(\frac{1}{1}\right)^{2n+1}}.(\cot 3\theta)^{\left(\frac{1}{3}\right)^{2n+1}}.(\tan 5\theta)^{\left(\frac{1}{5}\right)^{2n+1}}\ \&c=$$

$$\log^{-1}\left\{\frac{D^{-1}U(\pi)+D^{-1}U(-\frac{\pi}{2}-2\theta)-D^{-1}U\left(\frac{\pi}{2}-2\theta\right)}{2}\right\};(29).$$

These are but a few of the very singular results which may be deduced from our equation (12); but I shall forbear to extend this paper to an unnecessary length by any farther applications of it.

than unity; but when $n=0$, the series $S\left\{\frac{1}{x^{2n+1}}\right\}$ and $S\left\{\frac{1}{(2x-1)^{2n+1}}\right\}$ becoming infinite, they take the forms $\infty\times\log(1)$ which is altogether vague and inconclusive. Our equations (27) and (28), (29), then, are defective in this case, and we can only conclude that the function

$$(\tan\theta)^{\frac{1}{1}}.(\cot 3\theta)^{\frac{1}{3}}.\ \&c.$$

is independent on θ, or constant. There seems reason to conclude from other principles however that this constant is $=-\frac{\pi^2}{8}$, or more generally, $=(2i+1)\frac{\pi^2}{8}$ i being any integer, positive or negative.

III. *On Functional Equations.*

The determination of functions from given conditions is a point of such importance, not only in the partial differential calculus, but also in a variety of other branches, that it has occupied the attention of the most eminent Analysts, and it must be confessed, not without considerable success. Their researches, however, have hitherto extended no farther than to such conditions as involve only the unknown function, φ without any of its superior or inferior orders, φ^2, φ^3, . . . &c, φ^{-1}, &c. It is to equations of this latter kind, therefore, that we now propose to direct our attention.

The successive orders of any function $f(x)$ may be produced, either by actually writing $f(x)$ for x in the expression of $f(x)$, in which case the general value of $f^z(x)$ must be concluded from induction; or more elegantly by the following method.

Assume $f^z(x) = u_z$ and we have $f^{z+1}(x) = u_{z+1}$

$$0 = u_{z+1} - f(u_z)$$

an equation of differences whose integral will be of the form

$$u_z = \mathrm{F}(z, \mathrm{C})$$

C being an arbitrary quantity independent on z. Let $z = 0$, and we have

$$\mathrm{F}(0, \mathrm{C}) = u_0 = f^0(x) = x$$

an equation which gives C in functions of x.

For example; let $f(x) = 2x^2 - 1$, and we have

$$0 = u_{z+1} - 2u^2_z + 1$$

and integrating,

$$u_x = \tfrac{1}{2}\left\{C^{2^x} + C^{-2^x}\right\}$$

Now, if we make $u_o = x$, we get

$$C = x + \sqrt{x^2 - 1}$$

and consequently,

$$f^x(x) = \tfrac{1}{2}\left\{\left(x + \sqrt{x^2-1}\right)^{2^x} + \left(x - \sqrt{x^2-1}\right)^{2^x}\right\}.$$

If we suppose $\varphi(x, y)$ to denote any function of x and y, and conceive this expression substituted for x, as follows;

$$\varphi\left\{\varphi(x, y), y\right\},$$

we shall have the second *partial function*, taken with respect to x, which we may denote thus, $\varphi^{x,1}(x, y)$. If we repeat, or conceive repeated, this operation m times, we shall have the mth partial function with respect to x:

$$\varphi^{m,1}(x, y) = \varphi\left\{\varphi^{m-1,1}(x, y), y\right\}.$$

If the mth partial function with respect to x be in like manner successively substituted n times for y in the expression $\varphi(x, y)$, we shall obtain a result,

$$\varphi^{m,n}(x, y) = \varphi\left\{x, \varphi^{m,n-1}(x, y\right\},$$

and so on for more variables, z, w, &c. —. An equation containing any number of the successive orders $\varphi^0(x) = x$, $\varphi(x)$, ... $\varphi^n(x)$, of a function φ, and from which φ is to be determined, is called a functional equation of the nth order, in φ. Thus the equation

$$o = \varphi^2(x) - (1 + b)\cdot\varphi(x) + bx$$

is a functional equation of the second order, and is satisfied by the following

$$\varphi(x) = a + bx.$$

An equation between any number of the partial functions $\varphi^{m,n,\&c}(x, y, z, \&c)$ for determining the form of $\varphi(x, y, z, \&c)$ is called an equation of partial functions, and its order may be denoted in the same manner. Thus

$$0 = \varphi^{2,1}(x,y) + \varphi^{1,2}(x,y) - (a+b+1).\varphi(x,y) + c$$

is an equation of partial functions of the second order with two variables, and is satisfied by the equation

$$\varphi(x,y) = ax + by + c$$

Let $\varphi(x)$ be a function of x, and a certain number of constants $a, b, c, \ldots$ And from this expression conceive $\varphi^{a}(x)$, $\varphi^{\beta}(x)$ &c to be successively formed which will be functions also of a, b, &c. If the number of these constants be n, we may thus produce $n+1$ such functions of them, which will be respectively equal to the several orders of φ which they represent. Thus we have $n+1$ equations involving the n quantities $a, b, c,$ &c which may therefore be eliminated, and the resulting equation between x, $\varphi^{a}(x)$, $\varphi^{\beta}(x)$ &c will therefore be independent on them. As far then as regards this equation they are arbitrary, so that in reascending from a functional equation which contains $n+1$ different orders of φ (not including $\varphi^{0}(x)$ or x) n arbitrary constants must be introduced. The reasoning here made use of is sufficiently plausible, and in fact, no other than has been adduced in demonstration of well known and important truths. The conclusion too, to the extent of its literal meaning, is correct. But we have here to notice a paradox of a very singular nature, viz: that even in the simplest cases imaginable (such as $\varphi^{2}(x) = x$) the general expression for $\varphi(x)$ may contain, not one or two, but an unlimited number of arbitrary constants, nay

even one or more arbitrary functions. A nearer attention to every step of the above reasoning will explain this paradox. But what has been said will serve to make us cautious in trusting implicitly to all its other applications.

Problem I. To determine $\varphi(x)$ from the equation $\varphi^{s}(x) = x$. Assume z a function of x, and u a functional characteristic, which shall satisfy the following conditions

$$x = u_z, \varphi(x) = u_{z+1}.$$

From these, we obtain

$\varphi(x)$, that is, $\varphi(u_z)$ or $(\varphi u)_z = u_{z+1}$; . . . . . (*a*)

and $\varphi^{s}(x)$ or $\varphi\{\varphi(x)\} = (\varphi u)_{z+1} = x = u_z$; . . . . (*b*)

and, subtracting, $(\varphi u)_{z+1} - (\varphi u)_z = -(u_{z+1} - u_z)$

that is, $\Delta\{(\varphi u)_z + u_z\} = 0$; . . . . . (*c*)

and integrating,

$$0 = (\varphi u)_z + u_z + C.$$

Now by cross-multiplication of the equations (*a*) and (*b*) we find,

$$u_{z+1} \cdot (\varphi u)_{z+1} = u_z \cdot (\varphi u)_z.$$

Thus the function $u_z \cdot (\varphi u)_z$ does not vary when z changes to $z+1$, and of course must be considered as constant in the integration of (*c*). C therefore may be any function of $u_z \cdot (\varphi u)^z$, and thus our equation becomes

$$0 = u_z + (\varphi u)_z + f\{u_z \cdot \varphi(u_z)\}$$

or $0 = x + \varphi(x) + f\{x \cdot \varphi(x)\}$

an equation from which $\varphi(x)$ may be obtained for any assigned form of the function f. Thus if $f(x) = a + bx$,

$$o = a + x + (1 + bx) \cdot \varphi(x)$$

$$\text{and } \varphi(x) = -\frac{a+x}{1+bx}$$

which satisfies the condition proposed; and by giving f other forms, we should obtain other values of $\varphi(x)$.

The subsidiary function z, and the characteristic u are not then necessary to be *known* but as a matter of curiosity. They may however be found when φ is determined, by the resolution of the equation of differences $\varphi(u_z) = u_{z+1}$ which gives the form of the function u_z in z, and z is given by the equation $x = u_z$, or $z = u^{-1}(x)$.

Aliter. Assume as before, $x = u_z$, $\varphi(x) = u_{z+1}$ then we have

$$\varphi(u_{z+1}) = u_z; \quad \ldots\ldots\ldots\ldots\ldots\ldots (d).$$

Now, $x = u_z$ therefore $\varphi(x) = \varphi(u_z)$, that is, $u_{z+1} = \varphi(u_z)$ and for z writing $z-1$, $u_z = \varphi u_{z-1}$ which being substituted in (d) gives

$$\varphi(u_{z+1}) = \varphi(u_{z-1}).$$

Now this is a perfect function φ on both sides, and of course, taking the inverse function φ^{-1} on both sides

$$u_{z+1} = u_{z-1}$$

whence,

$$u_z = C\left\{\cos 2\pi z\right\} + (-1)^z \cdot C'\left\{\cos 2\pi z\right\}$$

C and C′ being two arbitrary functional characteristics. Now $u_z = x$, and consequently

$$x = C\left\{\cos 2\pi z\right\} + (-1)^z \cdot C'\left\{\cos 2\pi z\right\}.$$

From this conceive z found in functions of x, and call it $Z(x)$ then,

$u_{x+1} = \varphi(x) = C\left\{\cos 2\pi Z(x)\right\} - (-1)^{Z(x)} \cdot C\left\{\cos 2\pi Z(x)\right\}$. This method applies also to the more general equation $\varphi^s(x) = f(x)$, by the substitutions $f(x) = u_x$, $\varphi(x) = u_{x+1}$, but, owing to the transcendental equations it introduces, must be regarded as totally ineffectual and useless.

Prob. II. Given $\varphi^n(x) = f(x)$. Required at least one satisfactory value of $\varphi(x)$.

Let the general expression of $f^z(x)$ in functions of z and x found according to the method above explained be $F\{z, x\}$: we have then $\varphi^n = f$, and $\varphi = f^{\frac{1}{n}}$, that is

$$\varphi(x) = f^{\frac{1}{n}}(x) = F\left\{\tfrac{1}{n}, x\right\}.$$

Ex. 1. Let $f(x) = 2x^2 - 1$, or $\varphi^2(x) = 2x^2 - 1$, and we find

$$f^z(x) = \tfrac{1}{2}\left\{\left(x + \sqrt{x^2-1}\right)^{2^z} + \left(x - \sqrt{x^2-1}\right)^{2^z}\right\}$$

and of course

$$f^{\frac{1}{n}}(x) = \varphi(x) = \tfrac{1}{2}\left\{\left(x+\sqrt{x^2-1}\right)^{\sqrt[n]{2}} + \left(x-\sqrt{x^2-1}\right)^{\sqrt[n]{2}}\right\}.$$

We may here observe that *any one of the n values of* $\sqrt[n]{2}$ will equally afford a satisfactory value of $\varphi(x)$.

Ex. 2. Let $\varphi^n(x) = \frac{\alpha+\beta x}{\gamma+\delta x} = f(x)$

Assume $\varpi = \frac{\beta-\gamma}{2}$, $\lambda = \frac{\beta+\gamma}{2}$, $\mu = \left\{\varpi^2 + \alpha\delta\right\}^{\frac{1}{2}}$, $\nu = \frac{\lambda-\mu}{\lambda+\mu}$

and we shall find

$$f^{\frac{1}{n}}(x) \text{ or } \varphi(x) = \frac{\left\{\alpha + (\varpi-\mu)x\right\} \cdot \nu^{\frac{1}{n}} - \left\{\alpha + (\varpi+\mu)x\right\}}{\left\{\delta x - (\varpi+\mu)\right\} \cdot \nu^{\frac{1}{n}} - \left\{\delta x + (\varpi-\mu)\right\}}$$

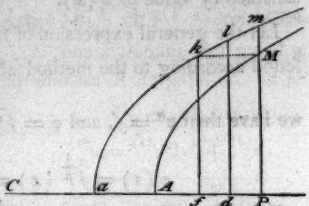
where any of the n values of $\nu^{\frac{1}{n}}$ may be taken, and thus as

many different values of $\phi(x)$ be obtained. This example depends on the integration of an equation of differences of the form

$$o = u_{x+1} \cdot u_x + A \cdot u_{x+1} + B \cdot u_x + C$$

a particular case of which had been previously integrated by LAPLACE in the Journal de l'Ecole Polytechnique.

Ex. 3. To take an instance of the application of these equations to geometrical problems, let AM be an hyperbola whose axis is CP and centre C, and let it be required to find a curve am such that drawing the ordinate PMm, making $Cd = Pm$ and again erecting dl parallel to PM, if this be repeated n times the last ordinate fk shall be equal to PM. Let $y = \phi(x)$ be the equation of am and $y^2 = (1 - e^2)(a^2 - x^2)$ that of AM, then $dl = \phi(Cd) = \phi(Pm) = \phi^2(x)$ and in like manner, $fk = \phi^n(x) = PM$, that is, $\phi^n(x) = f(x) = \sqrt{(1 - e^2)(a^2 - x^2)}$; consequently,



$$f^{\frac{1}{n}}(x) = \phi(x) = \left\{(e^2-1)^{\frac{1}{n}} \cdot x^2 - \frac{e^2-1}{e^2-2}\left\{(e^2-1)^{\frac{1}{n}} - 1\right\} \cdot a^2\right\}^{\frac{1}{2}}.$$

Thus we see, that am is also an hyperbola, whose centre is C, and calling a' and e' its semiaxis and excentricity, we have

$$e' = \sqrt{(e^2-1)^{\frac{2}{n}} + 1}, \text{ and } a' = a \cdot \left\{\frac{e^2-1}{e'^2-1} \cdot \frac{e'^2-2}{e^2-2}\right\}^{\frac{1}{2}}.$$

If AM be a right angled hyperbola, or $e = \sqrt{2}$, we shall have $e' = \sqrt{2}$ and $a' = \frac{a}{\sqrt{n}}$; that is, am is also a right angled hyperbola, having its axis $\frac{1}{\sqrt{n}}$ part of that of AM. If e

be < 1, and n odd, we shall have e' also < 1. Thus if the curve AM be an ellipse, am is also a concentric ellipse. The equations of *Ex.* 2. geometrically expressed afford a property of the hyperbola something similar.

IV. *On differential Equations of the first degree.*

Any equation of this species may be reduced to the form

$$o = u + {}^1\mathrm{A}\,.\,Du + {}^2\mathrm{A}\,.\,D^2u + \ldots . {}^n\mathrm{A}\,.\,D^nu + \mathrm{X}\,; \ldots (1)$$

u being the unknown, and ${}^1\mathrm{A}, \ldots . {}^n\mathrm{A}, \mathrm{X}$, known functions of x. To integrate it, assume the following equations

$$\left.\begin{aligned} u + {}^1\alpha Du &= u^{(1)} \\ u^{(1)} + {}^2\alpha Du^{(1)} &= u^{(2)} \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ u^{(n-2)} + {}^{n-1}\alpha Du^{(n-2)} &= u^{(n-1)} \\ u^{(n-1)} + {}^n\alpha Du^{(n-1)} + \mathrm{X} &= u^{(n)} = o \end{aligned}\right\}\ ; \ldots\ldots\ldots (2)$$

From these, eliminating successively $u^{(1)}, \ldots u^{(n-1)}$, we obtain

$$u^{(1)} = u + {}^1\alpha\,.\,Du$$

$$u^{(2)} = u + ({}^1\alpha + {}^2\alpha + {}^2\alpha D^1\alpha)\,.\,Du + {}^1\alpha\,.\,{}^2\alpha\,.\,D^2u$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$o = u^{(n)} = u + \left\{{}^1\alpha + {}^2\alpha(1 + D^1\alpha) + \&c\right\}\,.\,Du + \ldots\ldots$$

$${}^1\alpha\,.\,{}^2\alpha \ldots {}^n\alpha\,.\,D^nu + \mathrm{X}\,; \ldots\ldots (3)$$

The comparison of the coefficients of this equation with ${}^1\mathrm{A}, \ldots {}^n\mathrm{A}$, gives n equations for determining ${}^1\alpha, \ldots {}^n\alpha$, into which X does not enter. Consequently these functions are independent on X, and therefore, the same as if $\mathrm{X} = o$. Now the successive integration of (2) gives

$$u = \left\{\log^{-1}\int\frac{-dx}{{}^1\alpha}\right\}\cdot\int\frac{\log^{-1}\int\left(\frac{dx}{{}^1\alpha}-\frac{dx}{{}^2\alpha}\right)}{{}^1\alpha}\int \ldots\ldots\ldots$$

$$\int\frac{\log^{-1}\int\left(\frac{dx}{{}^{n-1}\alpha}-\frac{dx}{{}^n\alpha}\right)}{{}^{n-1}\alpha}\int\frac{-Xdx^n}{{}^n\alpha}\log^{-1}\int\frac{dx}{{}^n\alpha}; \quad \ldots\ldots (4)$$

which by writing o for X, and adding a constant at each integration becomes,

$$u = {}^1C\,.\log^{-1}\int\frac{-dx}{{}^1\alpha} + {}^2C\,.\left\{\log^{-1}\int\frac{-dx}{{}^1\alpha}\right\}\cdot\int\frac{\log^{-1}\int\left(\frac{dx}{{}^1\alpha}-\frac{dx}{{}^2\alpha}\right)}{{}^1\alpha}\,.\,dx + \&c. \quad (5)$$

Now, if ${}^{(1)}u, {}^{(2)}u, \ldots {}^{(n)}u$ be the particular integrals of

$$o = u + {}^1A\,.\,Du + \ldots\ldots {}^nA\,.\,D^nu; \ldots\ldots\ldots\ldots\ldots (6)$$

we shall have, when $X = o$,

$$u = {}^1C\,.\,{}^{(1)}u + {}^2C\,.\,{}^{(2)}u + {}^3C\,.\,{}^{(3)}u + \&c.$$

And comparing this with the expression (5), we find

$$\frac{-1}{{}^1\alpha} = D\log{}^{(1)}u;$$

$$\frac{-1}{{}^2\alpha} = D\log\left\{{}^1\alpha\,.\,{}^{(1)}u\,.\,D\frac{{}^{(2)}u}{{}^{(1)}u}\right\};$$

$$\frac{-1}{{}^3\alpha} = D\log\left\{{}^1\alpha\,.\,{}^2\alpha\,.\,{}^{(1)}u\,.\,D\frac{{}^{(2)}u}{{}^{(1)}u}\,.\,D\,\frac{D\frac{{}^{(3)}u}{{}^{(1)}u}}{D\frac{{}^{(2)}u}{{}^{(1)}u}}\right\};$$

$$\&c = \&c.$$

Suppose now that by any means whatever we can discover $n-1$ particular integrals ${}^{(1)}u, \ldots {}^{(n-1)}u$, of (6), the original equation deprived of its last term: then by the help of the first $n-1$ of these equations, the values of ${}^1\alpha, \ldots {}^{n-1}\alpha$, are given,

and from these, ${}^{n}\alpha$ may be derived by considering that the comparison of the equations (1) and (3) gives

$${}^{1}\alpha \cdot {}^{2}\alpha \cdot \ldots {}^{n-1}\alpha \cdot {}^{n}\alpha = {}^{n}\mathrm{A}, \text{ or } {}^{n}\alpha = {}^{n}\mathrm{A} \cdot ({}^{1}\alpha \ldots {}^{n-1}\alpha)^{-1}$$

Having obtained ${}^{1}\alpha, \ldots {}^{n}\alpha$, nothing more is requisite for obtaining a complete integral of (1), than to substitute their values in equation (4).

The method here delivered of obtaining the known theorems respecting the equation

$$o = u + {}^{1}\mathrm{A}\mathrm{D}u + \ldots {}^{n}\mathrm{A} \cdot \mathrm{D}^{n}u + \mathrm{X}$$

appears to have the advantage in point of conciseness over any I have hitherto met with; a sufficient apology for the revival of a subject whose theory, and whose difficulties have been so long and completely understood.

In the case when $\mathrm{X} = o$ and ${}^{1}\mathrm{A}, \ldots {}^{n}\mathrm{A}$ are constant, the method of separating the symbols of operation from those of quantity, may be introduced with great elegance.

Let p, q, r, &c be the roots of

$$o = \mathrm{D}^{n} + \frac{{}^{n-1}\mathrm{A}}{{}^{n}\mathrm{A}}\mathrm{D}^{n-1} + \ldots + \frac{1}{{}^{n}\mathrm{A}}$$

and the equation (1) becomes

$$o = (\mathrm{D} - p)(\mathrm{D} - q) \ldots \text{ \&c} : u$$

which is satisfied by either of the equations

$$o = (\mathrm{D} - p) : u, \qquad o = (\mathrm{D} - q) : u, \text{ \&c. or,}$$
$$\mathrm{D}u = pu, \qquad \mathrm{D}u = qu, \text{ \&c.}$$

Now these equations integrated give the following

$$u = \varepsilon^{px}, \quad u = \varepsilon^{qx}, \quad u = \varepsilon^{rx}, \text{ \&c.}$$

which are the particular integrals of the proposed, and of course its complete integral will be

$$u = {}^{1}\mathrm{C} \cdot \varepsilon^{px} + {}^{2}\mathrm{C} \cdot \varepsilon^{qx} + {}^{3}\mathrm{C} \cdot \varepsilon^{rx} + \text{ \&c.}$$

If there be m roots equal to p, we have

$$(D-p)^m : u = o, \text{ or } (D-p)^m : u \times \varepsilon^{-px} = o.$$

Now, $(D-p)^m : u \cdot \varepsilon^{-px} = D^m \left\{ u \cdot \varepsilon^{-px} \right\} = o.$

Therefore, integrating m times

$$u \cdot \varepsilon^{-px} = {}^{\circ}C + {}^{1}C \cdot x + \ldots . {}^{m-1}C \cdot x^{m-1}$$

and $$u = \left\{ {}^{\circ}C + {}^{1}Cx + \ldots . {}^{m-1}C \cdot x^{m-1} \right\} \varepsilon^{px}$$

which is the part of the integral arising from the equal roots p.

JOHN F. W. HERSCHEL.

London, Jan. 29, 1814.

XIII. *Observations on the Functions of the Brain.* By Sir Everard Home, *Bart. F. R. S.*

Read May 26, 1814.

THE various attempts which have been made to procure accurate information respecting the functions that belong to individual portions of the human brain, having been attended with very little success, it has occurred to me, that were anatomical surgeons to collect in one view all the appearances they had met with, in cases of injury to that organ, and the effects that such injuries produced upon its functions, a body of evidence might be formed, that would materially advance this highly important investigation.

For this purpose, I have brought together the following observations collected in the course of my professional pursuits, stating them, as so many experiments made upon the brain, with the conclusions which tend to elucidate this particular inquiry, referring to the notes, for fuller information.

The materials that can be furnished by an individual, are necessarily very small; they may however be sufficient to shew the advantages to be derived from this mode of investigation, and by that means connect still more closely the pursuits of anatomy with those of philosophy.

SECTION I.

The effects produced by an undue pressure of water upon the Brain.

Before I enter into the particular effects that take place when pressure is made upon the brain by means of water, it is necessary to mention that sudden pressure of any kind upon the cerebrum, takes away all sensibility, whether made upon the external surface through the medium of the dura mater,* or upon the internal parts through the medium of the ventricles, † and sensibility returns as soon as the unusual pressure is removed.

Faintness is the consequence of the pressure, to which the cerebrum has been accustomed, being suddenly taken off. ‡

I am induced to believe that pressure to a certain degree uniformly kept up, is necessary for the performance of the healthy functions of the cerebrum; and any increase or diminution of this pressure puts a stop to them. It is asserted, that in addition to this pressure, the pulsatory motion of the blood in the arteries of the cerebrum is also necessary; but the late JOHN HUNTER, whose accuracy in a point of this kind is not to be doubted, retained his senses although the heart had apparently ceased to act.§

* After the operation for the trepan, before the skull at that part is ossified, pressure upon the brain with the finger produces insensibility.

† This I have seen in cases of spina bifida, by pressing upon the tumour in the back, and forcing the water up into the ventricles.

‡ The removal of a coagulum of blood, half an inch thick, between the skull and dura mater, made the pulse nearly stop, but as soon as he recovered from this faint state, the person was perfectly well.

§ Vide Life of John Hunter prefixed to his work on the Blood, Inflammation, and Gun-shot Wounds. The account is transcribed from his own notes.

Although insensibility is the common effect of undue pressure upon the cerebrum, it appears from what will be stated, that it is not a necessary consequence of undue pressure upon the cerebellum.

The facts which have been stated appear to point out the use of the water in the ventricles of the brain, and they account for the great variety which is met with in the form and extent of the posterior cornua of the lateral ventricles, their size varying according to the quantity of water which is necessary to keep up the pressure required.

The size of the ventricles would appear to be very immaterial, since even when they are increased so as to contain above six ale pints, the functions of the brain are all carried on, and the growth of the body proceeds; but after the skull is completely ossified, an increase of two or three ounces produces insensibility.

That the ventricles should admit of being increased to so great an extent, without any of the senses or faculties of the brain being destroyed, is in itself a curious fact, and of so much importance with respect to the physiology of the brain, that I shall detail the two following cases, which illustrate one another.

In the one, the accumulation of water proceeded, as it will appear, as far as it could go without materially impairing the organ; it then stopt, and the boy grew up, with all his faculties: in the other the water continued to increase, the substance of the cerebrum was absorbed, and the faculties of the brain were destroyed.

A boy at a month old had so rapid an increase of the size of his head, as to evince an accumulation of water in the brain; and when he was five years old, the head was so large that the

parents, judging from recollection, believe that it never after increased. It was so transparent, that when exposed to the sun the rays passed through it as they would through a horn lantern. He was unable to walk. At this age, he caught the natural small-pox, which was so violent as nearly to prove fatal. Upon his recovery, the head shewed no disposition to increase, and the child in all respects began to improve, and for the first time learnt to walk. At fourteen, the skull appeared completely ossified. At nineteen years, the time I saw him, he was five feet six inches high; his head measured in circumference 33½ inches. He had grown in the course of the last year about two inches, which is more than he had usually done in any one year.

All the organs of sense are entire; savoury food is agreeable to his taste, but he is moderate in eating. His sight is good, but looking with attention at objects more than half an hour, appears to strain his eyes. His head is so heavy, that the muscles of the neck are unable to support it for many hours together: when he lies down, the head is supported by another person.

He sleeps with most ease on the right side, and the left side of the head appears to the eye to be rather the largest. In lying down, there is, what he describes to be, a momentary thrilling heat felt on the upper part of the brain, in the line of the longitudinal sinus. Lying on his back strains his eyes so much, that he cannot continue in that posture; stooping forwards brings an oppression upon his eyes. The least weight in his hand, as a tea cup, makes it tremble: all sudden noises jar his head, and produce giddiness. When he falls down, the jar renders him insensible; at one time this was the case for fifteen minutes, without being attended with any bad consequences.

His head aches when exposed to heat. He has had no illness since the small-pox. His sleep is easily broken: he never dreams. He is fond of reading and writing; has a taste for poetry, and can repeat verses out of COWPER. His memory of common things is very good. He never expressed any attachment or passion for women. He is of a mild disposition; but when irritated, his whole frame is in a state of agitation, which, however, soon goes off.

In another boy, the enlargement of the head was perceived at three months, and increased for three years; and then appeared to be stationary, and the child till that period was sensible. The upper part of the skull from that time began to ossify, and in three years more there was only an irregular space at the fontanelle, and a small space between the two portions of the os frontis remaining open. The child continued sensible till three years old, and then became gradually less so, did not know what it did; heard sounds, but could not see. At six years old he died.

The child was three feet three inches high, the skull twenty seven inches round; the water contained in the two lateral, and third ventricles was six ale pints and a half in quantity. The cerebrum formed a thin case of medullary substance surrounding this cavity. The cerebellum was entire.*

* The lining of the lateral ventricles was tough; the septum lucidum elongated so that the corpus callosum was raised up close to the skull, the falx of the dura mater being nearly obliterated. The water in the third ventricle had split the fornix and septum lucidum into two, and the thin membranes of the septum had holes in them, making a communication between the third and lateral ventricles. The substance of the brain surrounding those cavities, as well as the pia mater covering it, had no convolutions; there was one continued smooth surface. On the right side, upon which the child was usually laid, there were no remains of medullary or

	lb.	oz.	dr.
The weight of the whole brain was - -	2	8	1
The weight of the whole brain of a child between six and seven years - - -	2	12	0

The preceding facts explain satisfactorily that the cerebrum is made up of thin convolutions of medullary and cortical substance surrounding the two lateral ventricles; which are unfolded when the cavities of those ventricles are enlarged, and in this unfolded state the functions belonging to this part of the organ can be carried on.

Although the quantity of water may be so much increased without material injury to the functions of the brain when the skull is not ossified, yet after that period even a few ounces in the lateral ventricles has been known to produce so much undue pressure, as to bring on head-ache, general uneasiness, a sensation as if the head were too large, loss of spirits, convulsions, loss of memory of recent events, idiotism, insensibility, and death.*

cortical substance, and there the pia mater, and dura mater adhered together. There was no remaining brain between the third ventricle and sella turcica. On the left side of the left hemisphere the medullary and cortical substance was only half an inch thick. The corpora striata, and thalami nervorum opticorum were small and tough: the union between the thalami was elongated into a broad flat ligament. The two commissures, and iter ad infundibulum had the natural appearance. The pituitary gland had become flattened. The fourth ventricle, tuberculum annulare, and cerebellum, had nearly the natural appearance. The olfactory nerves were tough and small; the optic nerves had no medullary pulp; the other nerves going out of the skull had undergone no change.

* One ounce and half of water in the lateral ventricles, at 4½ years old, were accompanied by pain in the head, and great general irritability. Five ounces at 7 years by insensibility for 14 days before death. Two ounces and one half at 9 years by violent head-aches, an unwillingness to be moved, and stupor. Three ounces at 11 years were attended with loss of recollection of recent events. This began at the

When the water, instead of being contained in the general cavities of the lateral ventricles, is principally confined to their posterior and anterior cornua, the effects sometimes are occasional constipation, pain in the bowels, and lower part of the belly.* When accumulated in the third ventricle without any increase in the lateral ones, distressing pain in the head, loss of speech, and insensibility have occurred.† When accumulated in the ventricles of the brain, and also under the tuberculum annulare, painful sensations in the stomach, bowels, lower belly, and across the legs have been met with.‡ When not only in the ventricles, but between the tunica arachnoides and pia mater over the hemispheres, and also upon the tubercula quadrigemina, the apparent consequences in one

age of 10. A stupor of 12 days preceded death. Four ounces at 11 years were followed by the sensation of the head being too big, loss of speech, insensibility, convulsions. Eight ounces at 7½ years were accompanied by a state of idiotism for ten days, and then death.

* Five ounces of water contained principally in four small cells, two in the anterior and two in the posterior cornua of the lateral ventricles, at 6 years of age, were the only evident cause of pain in the lower part of the belly, occasional constipation, and violent pain in the bowels.

† Two ounces of water in the third ventricle, enlarged by the layers of the septum lucidum being separated, at 30 years of age, appeared to produce distressing pain in the head, loss of speech, and insensibility. Two drams of water in the third ventricle of an aged dog, were attended for four years by fits like apoplexy, pain in the head only relieved by opium, convulsions, and death.

‡ Two ounces of water in the lateral ventricles, ½ ounce under the tuberculum annulare between the tunica arachnoides and pia mater, at 5 years of age, were followed by painful sensations in the lower belly, stomach, and bowels.

Two ounces of water in the ventricles one under the tuberculum annulare, at three years appeared to produce not only painful sensations in the lower belly, stomach, and bowels, but pain in the head and pain across the legs, as if they were cut with an instrument.

case were depression of spirits, pain in the back of the head, and mania.* When in the ventricles, and also between the tunica arachnoides and pia mater, and between the dura and pia mater, melancholy, imbecility, apoplexy, and paralysis of one side, have been the accompanying symptoms.† When in the ventricles, where there has been an unusual vascularity of the dura mater, violent affections of the præcordia have occurred in the night during sleep, which have led to suicide.‡ When between the dura and pia mater in considerable quantity, a state of melancholy and imbecility of mind has been met with.§

* Water in the third ventricle in sufficient quantity to separate and keep apart the thalami nervorum opticorum, also between the tunica arachnoides and pia mater, over the hemispheres, and under the tubercula quadrigemina, in an adult, were attended by pain in the back of the head, low spirits, and after drinking wine, very high spirits. This ended in mania, and, after three months, in death.

† Two ounces in the lateral ventricles, one between the dura and pia mater; the space between the tunica arachnoides and pia mater loaded with it, the consequences of attacks of inflammation upon the coverings of the brain in an adult were attended by these symptoms. The paralysis took place after the last apoplectic fit, and death ensued.

‡ Two ounces in the lateral ventricles and a very unusually vascular state of the dura mater in an adult was followed by these effects.

§ A gentleman fell from a horse. From that time he had head-aches, gradually became melancholy and imbecile. He lived three years in that state. Four ounces of water were found between the dura and pia mater, on the right side. There was an exostosis $\frac{1}{4}$th of an inch long from the parietal bone with a sharp point, in contact with that part of the dura mater under which the water lay. There were four ounces in the lateral ventricles, the posterior cornua were very small, there were also three ounces on the basis of the skull.

SECTION II.

The effects produced by concussion of the Brain.

Concussion of the brain produces delirium and coma; these symptoms go off, they sometimes in a few days return and prove fatal.*

In the torpid state commonly attendant upon any violent shake being given to the brain, the senses are so much impaired that little information can be gained respecting the effects produced upon the internal organs. The bowels have been found under such circumstances to be acted on by aperient medicines with great difficulty.†

SECTION III.

The effects produced when the blood vessels of the Brain are preternaturally dilated or diseased.

Sudden dilatation of the blood-vessels of the cerebrum, in consequence of exposure to the sun, is sometimes accompanied by delirium; loss of speech and the power of swallowing.‡

A dilated state of the veins of the cerebrum has been attended

* This happened in two cases, and no appearances of alteration of structure in the brain were met with after death.

† A gentleman fell from his horse, and had a concussion of the brain. While in that state it required 60 grains of jalap and 20 of calomel to procure one evacuation from the bowels.

‡ In a case of coup de soleil in the West Indies these symptoms were produced, and the person died in two hours. The brain was examined four hours after death. The scalp felt hot, was loaded with blood; the cerebrum was hot to the feel; and a general distention of the blood vessels of the pia mater was the only unusual appearance, except that the substance of the cerebrum was unusually soft.

with head-aches, which are very severe when the body is placed in a horizontal posture.*

When the smaller arteries of the cerebrum are preternaturally enlarged while those of the cerebellum are not, delirium has taken place, followed by a fit resembling apoplexy, and a paralytic affection of one side.†

An obstruction to the passage of the blood through the right internal carotid artery was attended by a succession of slight apoplectic fits, unaccompanied by any paralytic affection.‡

An aneurismal enlargement of both the internal carotid arteries to the size of marbles projecting into the cavernous sinuses, was the only apparent cause of attacks of mania, with consciousness of being insane.§

* In the case of a young lady who had suffered severely from head-aches, which were so violent at night when she lay without a pillow as to produce occasional delirium, after death the veins in the medullary and cortical substance of the brain were found considerably enlarged.

† A man 60 years of age had from anxiety an apoplectic fit which lasted some hours, leaving a paralytic affection. In seven days, under a course of electricity, he recovered the use of the paralytic limbs. In four months, while under very great anxiety of mind, he became delirious, and one side became paralytic, in which state he died. There were no other appearances than those mentioned.

‡ An officer had a succession of apoplectic fits at intervals of one, two, and three months; the first of six days continuance, the second a few hours, the third and fourth shorter; the last terminated in death. On examining the brain, the right internal carotid artery was filled with a solid coagulum of blood which extended some way into the smaller branches.

§ A lady who had unequal spirits, and occasionally double vision, had an attack of giddiness and mania; the eyes were red, the hands benumbed, and death ensued. On examining the brain, the aneurisms were discovered; and the optic nerves were found wasted.

SECTION IV.

The effects produced by extravasated blood.

Blood in the lateral and third ventricles was attended by repeated fits of vomiting and coma.* In the fourth ventricle, a fit which in twenty four hours terminated in death;† under the anterior lobes of the brain by hiccoughs and stupor;‡ under the cerebellum, by convulsions of the neck and body, with drawing up of the feet without stupor;§ in the folds of the pia mater covering one hemisphere, by a paralytic affection of the opposite side, without any other symptom.‖

Blood in the folds of the pia mater over the posterior lobes of the brain, and serum in the cornua of both the lateral ventricles were attended by giddiness, paralysis, straight objects appearing crooked, loss of memory, and at last idiotism.¶ In the right thalamus nervi optici, extending into the lateral ventricles, by paralysis of the left side of the body, both eye-lids closed, the mouth drawn on one side, a perception of light

* A coagulum of blood, the size of a common leech, lying upon the plexus choroides of each lateral ventricle, and two ounces of serum were found in a person who had the symptoms stated.

† Half an ounce of blood was found in the fourth ventricle.

‡ Three ounces of coagulated blood were found upon the basis of the skull, under the anterior lobes.

§ Blood was found coagulated under the cerebellum.

‖ The extravasation was in small quantity.

¶ A person 69 years old had a paralytic stroke, from which he recovered; but had giddiness, and straight lines appeared crooked. He had a second, which terminated in idiotism which lasted a year and a half; he then died. It appeared that there had been two distinct extravasations of blood in the pia mater, which had communicated with the cornua of the lateral ventricles, in which were deposited six ounces of serum.

with the right eye, but not with the left, succeeded by coma.* Between the dura mater and skull covering the right hemisphere, by stupor which went off on its removal; but taking off the pressure produced faintness for a few minutes.†

Coagulable lymph spread over the union of the optic nerves, the pineal gland, and tuberculum annulare, was followed by permanent contraction of the muscles between the occiput and vertebræ of the neck, dilatation of the pupils, and a great degree of deafness.‡ Serum under the cerebellum by restlessness, convulsions, incessant talking, at times incoherent, and the eyes became insensible to light.§

SECTION V

The Effects produced by the formation of Pus.

Pus in the cornu of the right lateral ventricle was accompanied with delirium and convulsions. Under the tuberculum annulare by vomiting and delirium. Under the dura mater covering the right hemisphere, by delirium succeeded by coma. Under the left parietal bone, by watchfulness, sickness, irregular pulse, clammy sweats, talking incessantly. These effects went off on its removal: the quantity a tea spoonful.

* An ounce of blood was found in the substance of the right thalamus nervi optici extending into the right lateral ventricle; the left lateral ventricle was filled with bloody serum.

† The coagulum was about half an inch thick.

‡ There was a layer of coagulable lymph, extending over the sella turcica to the parts mentioned.

§ The quantity was two ounces.

SECTION VI.

The effects produced by depression, and thickening of different portions of the Skull.

Unusual pressure of the skull upon the middle lobe of the brain, was attended with pain in the stomach, torpor of the bowels, nausea, retching, pain between the shoulders, and in the feet.* On the upper part of the hemisphere, want of sleep, head-ache, and stupor. Those effects went off upon its removal.† On both of the anterior lobes of the brain, heaviness, loss of memory, depression of spirits, bordering on idiotism ‡ On the anterior lobes of the brain, accompanied with water between the tunica arachnoides and pia mater covering the superior part of the hemispheres, an apoplectic fit, heaviness, loss of memory, and a second apoplectic fit, which terminated in death.§ On the lower and lateral part of the left posterior lobe of the brain, uneasiness in the skin of the left cheek, extending along the chin, throat, and trachea, hissing noise in the ears, inability to speak the words

* The pressure was continued by a bony tumour in the form of a hemisphere, half an inch in thickness; the broad surface attached to the parietal bone.

† A depression of the under table of the left parietal bone close to the sagittal suture not more than an inch in length, and depressed for about one eighth of an inch.

‡ The frontal bone was increased in thickness to a considerable degree. The person had been many years resident in India.

§ There was a thickening of the os frontis, with small exostoses the size of peas upon its inner surface. The anterior lobe cerebri had a flattened appearance, and the tunica arachnoides was loaded with water.

the person wished to articulate, using others in their place, although conscious of doing so, and unable to correct it. Numbness in the arms and legs. These effects ceased on taking off the pressure.* On the anterior lobes of the brain, both anteriorly and laterally, with thickening of the pia mater, spasms in the lower extremities, and total loss of memory, so that the person did not know what he had done a few hours before: although in other respects in health.† On the lower and lateral portions of the anterior and middle lobes of the brain, head-aches, general wasting, irregularity in the action of the bowels; the feel of inability to swallow, and great distress in the act of swallowing, with great general irritability.‡

* A gentleman fractured his skull, and remained fifteen minutes insensible; became sensible but could not speak for seven days: for twenty eight days could not speak distinctly, and used one word for another. In six weeks he was considered well with a depression of the lower posterior part of the left parietal bone, two and a half inches long, one and a half broad, and three quarters deep, which was removed at the end of three years, the increase of symptoms making it necessary; they all went off.

† The frontal bone and the parietal ones were one third of an inch thick. There was an ossification in the falx of the dura mater near the crista galli, one inch and a half long, three quarters broad; another near the tentorium three quarters of an inch long, half an inch broad, and three quarters thick. The tunica arachnoides and pia mater thicker than the dura mater. The processes of the pia mater firm; the smaller arteries in the medullary substance carried red blood. Two ounces of water in the ventricles. The person had been thirty-five years in India.

‡ On the basis of the skull were numerous small exostoses, particularly from the lower portion of the parietal bones, some longer than others and sharp at the point; the longest, one third of an inch.

SECTION VII.

The effects of pressure from Tumours.

An hydatid imbedded in the substance of the right hemisphere of the brain, was attended with violent head-aches, and occasional fits similar to those of apoplexy.*

A tumour in the substance of the posterior lobe of the brain, was attended with derangement of the functions of the stomach and bowels, double vision, and afterwards loss of sight.† A tumour pressing on the left hemisphere; settled melancholy, drowsiness after dinner, requiring being carried into the air, which took it off, but it returned on coming back to the table.‡ A tumour in the fourth ventricle, epileptic fits, soreness in the throat, and great pain in the act of deglutition.§ A tumour in the tuberculum annulare and water in the ventricles, pain in the head, stumbling in walking, the mouth drawn on one side, loss of sight of one eye, although the pupils were not affected; dullness in hearing, difficulty of swallowing, so as to die starved, with all the mental faculties entire.‖

* It was the size of an orange, had firm coats in which was contained a limpid fluid in the quantity of four ounces. The sides of the lateral ventricles were closely pressed together.

† The tumour was of the steatomatous kind just above the tentorium, of the size of a turkey's egg, so as to raise up the posterior part of the lateral ventricle. The complaint, till just before death, was mistaken for worms.

‡ The tumour was of a soft nature, the size of a filbert, attached to the left side of the falx of the dura mater, a little above the tentorium, pressing upon the left hemisphere of the brain.

§ The tumour was of a soft steatomatous structure, the size of a walnut, but adapted in its shape to the form of the ventricle.

‖ The tumour was of the size of a walnut, composed of a suety matter, four ounces of water were found in the ventricles, and the tunica arachnoides was unusually dry.

SECTION VIII.

Effects of injury to the substance of the Brain.

A deep wound into the right anterior lobe of the brain, attended with inflammation and suppuration, produced no sensation whatever; the senses remained entire, and the person did not know that the head was injured.*

The brain shooting out in the form of fungus, after the dura mater is wounded, has no effect upon any of the nerves, nor is it attended with sensation;† but the inflamed pia mater gives great pain.

Loss of a portion of the medullary substance of the anterior lobe of the cerebrum, produced no symptoms.‡ Loss of a portion of one of the hemispheres was attended with difficulty of swallowing for twenty-four hours, and slight delirium of short duration.§ Ulceration of the anterior lobe of the brain, as low as the anterior cornu of the lateral ventricle, but not communicating with it, paralysis of both arms.‖

* From an explosion of gun-powder, a piece of copper three inches long was forced through the eye into the brain; but the arm being also wounded, the person never knew the eye to be injured. When suppuration came on he became insensible, and in a few hours died.

† A fungus from the brain shot out through the dura mater after the operation of the trepan, upon the left parietal bone. The pia mater surrounding it was so sensible, that when touched, the pain was excruciating.

‡ A boy five years of age, fractured his skull, and wounded the brain, half an ounce came away. When he grew up he was more acute, and had a better memory than his brother.

§ A fracture of the parietal bone, and wound of the dura mater, produced the injury to the brain.

‖ The ulceration took place in consequence of the trepan wounding the dura and pia mater.

In a case of a penetrating wound into the right hemisphere of the brain with bone forced into its substance, while there was an opening for the discharge of matter, no effects were produced, except when the circulation was much increased, and then only head-ache and numbness in the left side.*

SECTION IX.

Effects of alteration of structure in the Brain.

In a case in which the tuberculum annulare had undergone a change in its texture, and become so hard as with difficulty to be cut with a knife, a considerable quantity of earthy particles being intermixed with the medullary substance of the crura, and other parts of the cerebellum, and the cerebrum and upper part of the cerebellum unusually soft; the effects were, the boy had been an idiot from his birth, never walked, spoke, or understood what was said. Went often three days without food. At sixteen, when he died, was no bigger than a child three years old, except the head, which was as large as it is usually at twelve.†

SECTION. X.

Effects of injury to the Medulla Spinalis.

Pressure upon the medulla spinalis in the neck by coagulated blood produced paralytic affections of the arms and legs, all

* A musket-ball fractured and depressed a portion of the superior and posterior part of the parietal bone, but did not penetrate: an abscess formed in the substance of the brain in which the broken portions of bone were lodged. In this state the person was capable of doing his duty as a naval officer from China to England, but died in consequence of an attempt to extract the depressed bone.

† The cranium was not completely ossified, the fontanelle being still of a large size. This case was examined by Mr. Brodie.

the functions of the internal organs were carried on for thirty-five days, but the urine and stools passed involuntarily.*

Blood extravasated in the central part of the medulla in the neck, was attended with paralytic affection of the legs, but not of the arms.†

In a case where the substance of the medulla was lacerated in the neck, there was paralysis in all the parts below the laceration; the lining of the œsophagus was so sensible, that solids could not be swallowed, on account of the pain they occasioned.‡

Where the medulla in the back was completely divided, there was momentary loss of sight, loss of memory for fifteen minutes, and permanent insensibility in all the lower parts of the body. The skin above the division of the spinal marrow perspired, that below did not. The wounded spinal marrow appeared to be extremely sensible.§

* A coagulum of blood, the thickness of a crown piece, was found lying upon the external surface of the dura-matral covering of the medulla spinalis, extending from the fourth vertebra colli, to the second vertebra dorsi. The medulla spinalis itself was uninjured.

† The sixth and seventh vertebræ colli were dislocated; the medulla spinalis externally was uninjured; but in the centre of its substance, just at that part, there was a coagulum of blood nearly two inches in length.

‡ The seventh vertebra colli was fractured, and the medulla spinalis passing through it, was lacerated and compressed.

§ The spinal marrow within the canal of the sixth vertebra dorsi was completely destroyed by a musket ball. The person lived four days.

XXIV. *Further Experiments and Observations on Iodine.* By *Sir* H. Davy, *LL. D. F. R. S. V. P. R. I.*

Read June 16, 1814.

1. *On the triple Compounds containing Iodine and Oxygene.*

1. In this communication I shall have the honour of presenting to the Royal Society, a continuation of the inquiries I have made respecting the chemical agencies of iodine, and the properties of certain of its compounds.

2. I described in my last paper the action of iodine on fixed alkaline lixivia, and the deflagrating salts it forms. In the first experiment which I made on these compounds, I employed the first crystals which fall down from moderately strong solutions of potassa and soda saturated with iodine, which had been purified by being repeatedly acted upon by distilled water: I now find that this process is not sufficient to free the triple compound from the double compound; and that to obtain them in a state of absolute purity, it is necessary to boil them repeatedly in small quantities of alcohol of specific gravity of from 8.6 to 9.2, which dissolves the double compound, but has little power of action on the triple compound.

The triple compounds, when purified, present some curious chemical phenomena, which a minute quantity of the double compound adhering to the crystals that I operated upon, prevented me from observing in the experiments I have already communicated to the Society. I shall describe these

phenomena as they are produced by the triple compound of potassium, as this substance is most easily procured in considerable quantities, but as far as I have been able to observe, the phenomena presented by the compound of sodium are precisely analogous.

The triple compound of potassium purified by alcohol is almost tasteless, has no action on vegetable colours, is very little soluble in cold water, but more soluble in hot water; when it is thrown into concentrated nitric, or sulphuric, or phosphoric acids, it has no violent action on them. By heat it may be dissolved in them, and the solutions, when saturated, congeal and form crystalline substances intensely acid. When the substance formed by the triple compound and the nitric acid is strongly heated, the nitric acid flies off, and at the temperature at which it is entirely expelled, the substance itself begins to decompose and affords a little iodine and much oxygene.

If the solution of the triple compounds in sulphuric or phosphoric acids be heated strongly at the temperature at which the acids sublime, the triple compound itself is decomposed, and it affords oxygene and iodine, and leaves acid sulphate and phosphate of potassa. If when the mixture is rendered fluid by heat, a little sugar or other combustible matter is added, there is a violent action, and iodine is disengaged with great rapidity.

The triple compound dissolves without decomposition in solution of phosphorous acid; but on heating the solution, oxygene is attracted by it, iodine appears, and phosphate of potassa is formed.

When the triple compound is thrown into concentrated

muriatic acid, there is an effervescence, the smell of chlorine is perceived, the fluid becomes yellow, and when evaporated yields the chlorionic acid.

When the solution of the hydroionic acid in water is poured upon the triple salt, iodine is instantly produced in great quantities.

Acetic and oxalic acids dissolve the triple compound without decomposing it. On heating the solution in oxalic acid, the acid becomes brown from the deposition of charcoal, and iodine immediately appears.

When the triple compound is thrown into solution of sulphurous acid, iodine is instantly produced, and sulphuric acid formed, and if the sulphurous acid is not in too large a proportion, the solution becomes yellow by dissolving iodine; if more sulphurous acid is added, water is decomposed, and sulphuric acid, and hydroionic acid formed.

The *double compound* of potassium and iodine has no action on oxalic, acetic, sulphurous, or phosphorous acids, but when it is mixed with the triple compound it is instantly decomposed by them, and iodine set free.

The same double compound in its pure state is decomposed very slowly by muriatic acid; and to convert the greater portion into muriate of potassa (potassane) it is necessary that the acid should be very frequently distilled from it, and a part always remains unaltered; when mixtures of the triple and double compounds are exposed to the action of muriatic acid, potassane (muriate of potassa) is instantly formed; and if the proper proportions are adopted, none of the double or triple compounds remain, and the results are potassane only and the oxy-chloric acid.

Mixtures of the triple and double compounds produce abundance of iodine when acted on by glacial hydrophosphoric acid gas, but the pure double compound affords only hydroionic acid gas, and this decomposition offers the best method which has yet occurred to me of procuring pure hydroionic acid. When the two substances are gently heated together, the hydroionic acid gas, which comes over in considerable quantities, forms a colourless solution when absorbed by water.

I have endeavoured to ascertain the composition of the triple compound of potassium. Seven grains that had been dried at the temperature of boiling water heated to redness in a small crucible of platinum lost 2.2 grains. Seven grains heated to dull redness in a small tube of glass lost 1.7 grain; a minute portion of iodine condensed in the middle part of the tube, but no violet vapour was observed in the upper part of it, and there was a very slight appearance only of moisture, so that the loss of weight in this last experiment, must be principally ascribed to the expulsion of oxygene.

On a comparison of the results of these two analyses, it appears very probable that this triple compound is composed of one proportion of iodine about 163, one of potassium 75, and six of oxygene 90; which is a composition exactly analogous to that of the hyper-oxymuriate of potassa. The quantities that I used in my experiments were too small to render these results more than approximations, yet the similarity of them to those presented by the hyper-oxymuriates ought perhaps to render them more worthy of confidence.

I have attempted to obtain pure triple compounds from solutions of baryta and lime, and from magnesia diffused through water, by dissolving iodine in them by heat, and by

evaporating the clear liquor until it began to deposit crystals. In this way I have procured substances which, when well washed in distilled water, afforded no iodine to nitric acid, which yielded chlorine and chlorionic acid when acted upon by muriatic acid, and which when distilled afforded much oxygene and some iodine, and left substances which appeared to be mixtures of the earths with compounds that afforded iodine to sulphuric acid, producing a smell of sulphurous acid gas, and which probably consisted of the metals of the earths united to iodine.

The triple compounds from lime and magnesia were soluble without affording iodine in sulphuric acid; but on evaporating the acid, at the time that the vessel of platinum in which the experiment was made became dry and almost red hot, the violet vapour was perceived. Even the triple compound from baryta did not afford iodine or oxygene by treatment with sulphuric acid, except under the same circumstances.

8. When I first discovered that the triple compounds dissolved in acids without effervescence, I thought it probable that the effect depended upon the formation of a compound of oxygene and iodine, similar to euchlorine, or the oxychloric acid, and which remained dissolved in the fluid; and on this idea I made a number of experiments with the hope of obtaining such a combination in a detached form.

I distilled the solution of the triple compound of potassium in sulphuric acid, but the only gaseous product I obtained was oxygene. Sulphuric acid and iodine condensed in the cool part of the apparatus, and the residuum was acid sulphate of potassa.

Conceiving that a compound of oxygene and iodine might

nevertheless exist in the fluid, and be decomposable at a high temperature, I attempted to obtain it by acting on the triple compound of barium by sulphuric acid, and by evaporating the fluid obtained at a gentle heat, and suffering it to cool at different periods of the process; but in this manner of operating I gained no better results.

The triple compound of barium is scarcely soluble in water. Water that had been boiled upon it gave only a slight cloudiness to sulphuric acid, which possibly might be owing to some double compound mixed with it: the fluid when evaporated nearly to dryness afforded fumes which had the characters of those of sulphuric acid, and by a red heat yielded iodine, and left sulphate of baryta.

When the solid triple compound of baryta was heated in very small quantities of diluted sulphuric acid, the fluid separated exhibited acid properties, and when gradually evaporated left a substance which congealed by cooling, and formed a solid of a yellow colour deliquescent in the air, strongly acid, and which reddened vegetable blues, and did not afterwards destroy them. When strongly heated, the substance afforded the same results as the substance procured from the fluid just mentioned.

The residual solid matter obtained by the action of sulphuric acid on the triple compound of barium was treated a second time with sulphuric acid, yet notwithstanding, when heated to redness, it yielded iodine in abundance.

I have repeated these experiments very often, because M. GAY LUSSAC has stated that an acid compound of oxygene and iodine may be procured by dropping sulphuric acid into a solution of the triple compound of barium; but the conclusions

of this ingenious chemist seem to have been founded upon the want of effervescence in the process; and his experiments were made at a very early period of the investigation, and probably before this time he has found reason to alter his opinion.

It is probable that a binary compound of iodine and oxygene may be formed, but the facts presented by the action of acids on the triple compounds are not sufficient to prove its existence.

When small quantities of very diluted sulphuric acid are digested on the triple compounds of potassium and barium, the fluid obtained is always acid, and always precipitates muriate of baryta. I thought it possible that the compound of iodine and oxygene might possess this property; but on collecting the precipitate and examining it, it appeared to be a mixture of the triple compound and sulphate of baryta, and from all the facts it appears that in the action of acids on the triple compounds new combinations only are formed.*

I take the liberty of proposing for the triple compounds the names of oxy-iodes, because, when decomposed by heat, they

* When sulphuric acid is made to dissolve as much of the triple compound of potassium by heat as possible, the mixture congeals by cooling into a yellow transparent substance, extremely deliquescent, and very acid. On decomposing it by heat, *neutral* sulphate of potassa remains. Now as oxygene and iodine are the only substances driven off by heat, it may be asserted that the acid property of the mixture depends upon these two principles; yet this conclusion does not follow according to sound chemical logic: iodine alone destroys the alkaline properties of potassa, and oxygene and iodine in combination with potassium form a difficultly soluble and almost tasteless substance. This substance, the triple compound, has only a weak attraction for sulphuric acid, and it might be expected that in combining with sulphuric acid it would not deprive it of its acid properties.

afford oxygene and iodine. Individually they may be named from their bases. Thus *oxy-potassame*, or oxy-iode of potassium, will signify the triple compound of potassium, oxygene, and iodine, and *oxy-barame*, or oxy-iode of barium, will denote the triple compound of barium.

3. *Some observations on Hydroionic Acid, and on the compounds procured by means of it.*

1. I have generally procured the hydroionic acid which I have used in my experiments by the process referred to in the last section, the action of hydrophosphoric acid on potassame, but I have sometimes employed the gas procured from moistened iodine by phosphorus.

The hydroionic acid gas is rapidly decomposed by being heated in contact with oxygene, and a solution of iodine and hydroionic gas in water is formed, and it is slowly decomposed by heat alone, affording a deep red-brown easily fusible substance, which seems to be a compound of hydroionic gas and iodine.

When condensed in water, it is instantly decomposed by solution of nitric acid and iodine precipitated.

The solution of hydroionic acid rapidly absorbs oxygene from the air, and becomes yellow, and at last deep tawny orange; and this absorption is assisted by light and heat, the hydrogene is attracted by the oxygene to form water, and the iodine formed is dissolved in the remaining acid.

The concentrated hydroionic acid will probably form a good eudiometrical substance; it does not render the vessels in

which it is used cloudy like the hydrosulphurets by the deposition of solid matter, and it does not enlarge the volume of the residual air like some other substances.

The solution of the hydroionic acid is decomposed by being heated with the hyperoxymuriate of potassa, and iodine is produced.

Hydroionic acid gas, as I have mentioned in my last paper, is decomposed by all the metals I have exposed it to, except gold and platinum; and the same metals that decompose it in its gaseous state, likewise decompose it when it is in solution, requiring, however, in some cases, the assistance of heat. The fluid hydroionic acid tarnishes silver at common temperatures, and dissolves mercury slowly when boiled in contact with it.

2. It dissolves the alkaline and common earths, and forms with them compounds very analogous in their properties to the compounds they produce when acted on by muriatic acid.

I heated dry quick lime in a small tube filled with hydroionic acid gas, a yellow fluid immediately formed, which was coloured by dissolving hydroionic gas and iodine, and a fusible compound soluble in water, and which had a bitter taste similar to muriate of lime, was produced.

I made the same compound by dissolving marble in the hydroionic acid; the compound when heated to redness became fluid, and when kept in fusion in contact with air emitted iodine, gradually lost its fusibility, and from being neutral became alkaline, so that at a high temperature iodine is partly expelled from calcium by oxygene. I proved this still more distinctly by fusing the compound in a close vessel, in which it was confined by mercury. There was no change. I ad-

mitted a little oxy-potassame, and caused it to give off oxygene by heating it: as soon as the calcareous compound was fused in contact with oxygene, it instantly emitted iodine, and lime was formed on the surface.

The compound formed from hydroionic acid and baryta is an acrid bitter substance, very similar in its taste to *barytane*, (fused muriate of baryta,) not decomposable when heated to whiteness unless oxygene is present, but when it is heated in contact with oxygene, oxygene is absorbed, and a part of its iodine expelled.

Magnesia dissolved in hydroionic acid without effervescence, and the solution evaporated gave a solid substance, having a taste very similar to muriate of magnesia. Like that salt, it partly lost its acid by a red heat; but a portion remained not decomposable out of the contact of air, but which instantly afforded iodine when heated in contact with oxygene.

I dissolved glucina, ittria, and zircona in the hydroionic acid; they formed neutral saline compounds. The compound of hydroionic acid and glucina was less soluble and more astringent in taste than the muriate of glucina, and was entirely decomposed when heated in the open air, affording hydroionic acid and iodine.

The compound formed from ittria was more soluble, and highly astringent; that formed from zircona astringent, with more of bitterness. Both these salts were decomposed when heated in the atmosphere at a low red heat; a smell of hydroionic gas was perceived, iodine was produced, and the earths remained.

8. I mentioned, in a note dated Montpellier, Jan. 10, (containing a correction for my last communication to the Society,)

that the alkaline property which I at first supposed to belong to the compounds of potassium and sodium with iodine, depended upon some undecomposed subcarbonate of potassa mixed with the hydrate of potassa I employed, as the subcarbonate of potassa is decomposed by iodine and carbonic acid set free, I had not thought it probable that the subcarbonate of potassa could interfere with this result. But I find that if the subcarbonate exist at all in the lixivium, a portion of it always remains undecomposed. I find likewise, that when a solution of iodine in lixivium of potassa is rendered perfectly neutral, or even slightly acid by hydroionic acid, a strong red heat renders the solid substance obtained slightly alkaline, provided it be in contact with air. Whether the separation of iodine by oxygene, in this instance, depends upon some effect of the moisture contained in the atmosphere, or upon the continued action of fresh portions of oxygene on the same surface of the compound, it is not easy to say; but a similar effect I find is produced upon *potassile*, (fluate of potassa;) this substance gains the power of reddening paper tinged with turmeric, by being strongly heated in contact with the air.

The power of neutralizing acids does not belong to the true compound of iodine and potassium, but depends either upon the subcarbonate not decomposed, or upon the alkali formed during the ignition of the compound; the pure double compound seems to have no power of action on the acids it does not decompose; I fused it in contact with sulphurous acid gas confined by mercury in a glass tube, the salt gained a slight tint of yellow, but did not absorb its own volume of gas: after this, it slightly reddened litmus, so that the acid must have had little more than a mechanical adhesion to the salt.

When *potassame*, or iode of potassium, is fused with boracic acid, there is a perfect mixture of the two bodies. In my first researches on this mixture, I conceived that they entered into chemical union, and formed a violet coloured glass, and that the acid property of the boracic acid was neutralized by the new compound; but I since find that the violet colour of the glass is owing to the developement of iodine, and when the application of heat is long continued, much iodine is disengaged, and the colour of the glass changes to olive, and borate of potassa is formed. When the glass is dissolved in warm water, an olive coloured powder separates, soluble when boiled in the caustic alkalies, so that there is great reason to suppose that it is boron, and that the boracic acid is decomposed by the attraction of the potassium combined with the iodine for oxygene, assisted by the tendency of iodine to assume the elastic state.

I fused the neutral compound of iodine with silica; no change was effected when the experiment was made in close vessels, but when the mixture was exposed to air, and intensely heated, a little iodine was evolved, some potassa formed, and some silica dissolved by it.

3. *On other acid compounds of Iodine.*

1. I have made several experiments on the combination of iodine and chlorine, obtained by admitting chlorine in excess to known quantities of iodine in vessels exhausted of air, and repeatedly heating the sublimate.

Operating in this way, I find that iodine absorbs less than one third of its weight of chlorine.

The compound of iodine and chlorine is a very volatile substance, and in consequence of its action upon mercury, I have

not been able to determine the elastic force of its vapour. Hence the estimations of its composition from experiments on the quantity of chlorine absorbed in close vessels must necessarily be liable to error. In one experiment, in which I dissolved the sublimate, by admitting a small quantity of water into the retort, I found that eight grains of iodine had caused the disappearance of five and a quarter cubical inches of chlorine.

In another experiment, in which the sublimate was not dissolved by water, and in which the absorption was judged of by the admission of fresh quantities of the gas, twenty grains caused the disappearance of 9,6 cubical inches of chlorine, the barometer being at 30.1, and the thermometer at 57 FAHRENHEIT.

It seems probable, from these experiments, that the chlorionic acid consists of one proportion of iodine and one of chlorine.

The chlorionic acid formed by the sublimation of iodine in chlorine in great excess is of a bright yellow colour, when fused it becomes of a deep orange, and when rendered elastic it forms a deep orange coloured gas. It is capable of combining with much iodine when they are heated together, its colour becomes in consequence deeper, and the chlorionic acid and the iodine rise together in the elastic state.

The solution of the chlorionic acid in water likewise dissolves large quantities of iodine, so that it is possible to obtain a fluid containing very different proportions of iodine and chlorine.

The pure solution of the chlorionic acid, when it is very diluted, loses its colour by being agitated for some time in contact with chlorine, and then, when poured into fixed alkaline lixivia or solutions of the alkaline earths, it causes the

precipitation of substances having the characters of triple compounds or the oxyiodes. If it is coloured, or in its ordinary state, at the same time that the oxyiode is precipitated, much iodine appears, and it is impossible to render a *concentrated* solution colourless by agitation with chlorine, or to deprive it of its power of yielding iodine by acting on alkaline solutions. The chlorionic acid, when poured into a solution of muriate of baryta, causes a copious precipitate in it, which has all the characters of oxyiode of barium, and the liquor becomes very acid.

When the colourless solution of chlorionic acid is added to a strong solution of ammonia, a white powder is precipitated which detonates feebly by a gentle heat; and which, when decomposed in glass vessels, affords iodine and elastic matter which does not support combustion.

When highly coloured chlorionic acid is employed, the powder that falls down is black, and detonates with much greater force, and by the slightest touch or motion, and it appears to be the same substance as that procured directly by the action of iodine on ammonia, and which I have shewn to be a compound of azote and iodine. Whether the white powder is a similar substance containing a larger proportion of azote, or whether it is a compound of ammonia with oxygene and iodine, or with iodine and chlorine, I have not yet been able to determine.

It is soluble in excess of chlorionic acid, and in this way may be separated from the black powder; it affords a little moisture during its detonation, but it is not possible to say whether this is formed in the process, or whether it is water adhering to the compound, for the temperature of its decomposition is so

low, that a proper degree of heat cannot be applied to render it dry.

When two bodies so similar in their characters and in the compounds they form, as iodine and chlorine, act upon substances at the same time, it is difficult to form a judgment of the different parts that they play in the new chemical arrangements produced. When I found that the chlorionic compound formed a strong acid by solution in water, I at first suspected that water was decomposed and hydroionic acid and euchlorine formed; there was no effervescence in the process, and the proportions agree with the supposition; but I find that solution of euchlorine instantly decomposes hydroionic acid and precipitates iodine, which is afterwards redissolved by the chlorine set free; and nitric acid, which decomposes hydroionic acid, has no action on chlorionic acid.

It was possible likewise that if water was decomposed, muriatic acid and a compound of iodine and oxygene might exist in the solution; I endeavoured to ascertain if this was the case by distilling the solution at different temperatures and collecting the products, but I obtained always the same fluid.

When coloured solution of chlorionic acid is boiled with hyper-oxymuriate of potassa, it loses its colour, and chlorine is given off from it; but in this case it likewise gradually loses its acidity, and a substance which yields iodine by heat with much effervescence, and which is probably oxypotassame, is precipitated.

It appears to me most probable that the acid property of the chlorionic compound depends upon the combination of the two bodies; and its action upon solutions of the alkalies and the earths may be easily explained, when it is considered that chlorine has a greater tendency than iodine to form double

compounds with the metals, and that iodine has a greater tendency than chlorine to form triple compounds with oxygene and the metals.

When in the case of the action of the chlorionic compound on fixed alkaline lixivia the chlorine is not in great excess, much iodine is always set free, because as it is easy to perceive from the proportions in which they combine, there is not sufficient oxygene detached from the alkali by chlorine to form the triple compound; and if the estimation of the composition of oxypotassame given in the first section be accurate, supposing that none of the double compound of iodine is formed, a solution must contain five proportions of chlorine to one of iodine, to produce a triple compound without the precipitation of iodine. It is however most probable that some double compound of iodine is always formed, as a solution must be extremely diluted indeed to contain five proportions of chlorine to one of iodine.

When the solution of chlorionic acid is poured into solution of muriate of baryta, water must be decomposed to furnish hydrogene to the muriatic acid, and oxygene to the triple compound, and in this case some double compound of iodine and barium must be formed and remain dissolved in the solution.

From the action of chlorionic acid on metallic solutions, I am inclined to believe that triple compounds of the common metals, oxygene and iodine may be formed by means of it. It occasions a copious precipitate without effervescence in the solution of sulphate of iron, and in the solutions of nitro-muriate of lead, and tin, and of nitrate of copper, and from analogy it is probable that these precipitates consist of the metal, oxygene, and iodine.

2. I thought it probable from the rapid action of tin on iodine, that tin foil would burn in the vapour of iodine, but on introducing it into the violet coloured gas in a small retort made very hot, though the combination was instantaneous, yet no light was apparent.

I thought it possible that the acid properties of the compound of tin and iodine, which I have described in my last communication to the Society, might depend upon the decomposition of water and upon the formation of hydroionic acid. On this idea I distilled the solution of it in water, hoping if hydroionic acid were formed, that I should obtain some in this process; but the fluid that came over was merely water coloured by a minute quantity of iodine, and the orange coloured substance which remained when dissolved in water, exhibited the same acid properties as before, and combined with ammonia without affording any oxide.

The compound of iodine and iron when dissolved in water exhibited acid properties, but when the solution was distilled it yielded hydroionic acid and deposited oxide of iron, and the entire solution acted on by ammonia, afforded an olive coloured precipitate in great abundance.

4. *On the action of some compound gases on Iodine.*

1. I heated some iodine in a dry glass globe filled with sulphuretted hydrogene; there was a considerable absorption of gas, no sulphur was deposited, and a reddish brown fluid was formed, which when thrown into water rendered it strongly acid and deposited much sulphur; the water passed through a filter exhibited the properties of hydroionic acid.

It is evident from this experiment, that sulphur, iodine, and hydrogene, are capable of forming a triple compound.

2. I sublimed some iodine in dry olefiant gas; a little of a reddish brown fluid was formed, but the greatest part of the iodine crystallised on the sides of the vessel in which the experiment was made. By repeating the process several times, more of the fluid was formed. It was volatile at a moderate heat, and gave a yellow tint to water, but did not render it acid, there was a very slight absorption of the gas.

3. Iodine sublimed in nitrous gas effected no change in it.

4. When iodine was exposed to carbonic oxide it underwent no change, it was repeatedly sublimed in it in common day light without undergoing the slightest alteration.

When the violet gas was formed by heating iodine in carbonic oxide, and the vessel exposed for some time to the direct solar rays, a small quantity of a limpid fluid which had an acrid taste formed in the vessel. An accident prevented me from ascertaining if any gas had been absorbed, but it seems probable from this result that, like chlorine, iodine may be combined with carbonic oxide by the agency of light.

5. *On the mode of detecting Iodine in combinations, and on certain properties of its compound with Sodium.*

1. I have examined many of the marine productions of the Mediterranean, with the view of determining whether they contained iodine. The ashes of the fuci and ulvæ of this sea afford it in much smaller quantities than the *sel de varec*, and in a very few cases only have I been able to obtain evidences of its existence in them.

2. M. Berard was so good as to order a considerable quantity of the species of ulva, which abounds on the coast of Languedoc, to be burnt for me at his laboratory at Montpellier. The ashes consisted for the most part of common salt, but a small quantity of alkaline lixivium which was obtained from them, afforded a red fluid when acted upon by sulphuric acid, and a similar colour I found was produced, when a solution of subcarbonate of soda and common salt, containing a minute quantity of the compound of sodium and iodine, was treated in the same manner by the acid.

3. One of the best tests of the presence of a very minute quantity of iodine in compounds, is their action upon silver. Water when it contains less than $\frac{1}{1000}$ part of its weight of the double or triple alkaline compounds of iodine tarnishes polished silver.

The effect produced by compounds of iodine, may be distinguished from that produced by sulphurets or sulphuretted hydrogene by this circumstance, that solutions containing sulphurets or sulphuretted hydrogene, by being boiled with a little muriatic acid, no longer tarnish the metal, whereas solutions containing iodine still retain the power.

4. Amongst a number of sea weeds that were obligingly given me for examination, by professor Viviani of Genoa, the ashes of the following afforded slight indications of the presence of iodine,

Fucus cartilagineus. Fucus filamentosus.
—— membranaceus. Ulva pavonia.
—— rubens. —— linza.

In the ashes of the corallines and sponges, I could discover no evidences of the presence of the substance.

5. I have examined three specimens of alkali formed by the combustion of vegetables that grow on the sea shore, one from Sicily, one from Spain, and the third from the Roman states, but not one of them afforded any indications of the presence of iodine.

6. I evaporated a considerable quantity of sea water procured at Sestri of Levanto in Liguria, in a part of the bay remote from any source of fresh water; but I could gain no unequivocal evidences of the presence of the compounds of iodine in it. The residual liquor after the common salt had been separated, did not act upon silver nor colour sulphuric acid. The first crystals of salt which fell down when fused upon silver, appeared to me to tarnish it more than the last; from which it appeared probable that they may have contained some triple compound of iodine, yet after being ignited, they did not colour sulphuric acid. When a large quantity of this water was electrised by a Voltaic apparatus, and the products separated at the positive pole collected in a small cup of gold, which was covered with cement, except in the interior and lower part forming the circuit, a yellow solution was obtained, which, when it was exposed to the negative pole of a Voltaic apparatus, yielded a black powder fixed in the fire, and not unlike the compound formed by heating gold and iodine together; but the quantity was too minute to admit of analysis, and a dark coloured substance is likewise obtained by negatively electrifying oxymuriate of gold, and there can be no doubt but that this substance formed a principal part of the solution.*

* Iodine, like chlorine, I find combines both with gold and platinum, when heated with them, or when they are exposed to them in its nascent state.

If iodine exists in sea water, which there is every reason to believe must be the case, though in extremely minute quantities, it is probably in triple union with oxygene and sodium, and in this case it must separate with the first crystals of common salt.

Whether the superiority which the curers of fish and meat are in the habit of attributing to bay salt over rock salt, is at all connected with the presence of the compounds of iodine, is an inquiry perhaps worth making, and the results of Dr. HENRY's elaborate investigation of the composition of different kinds of salts, do not preclude the possibility of the circumstance, though they certainly diminish the probability.

I rubbed pieces of beef that had been killed some days, with the double and triple compounds of sodium. They did not putrify; the one rubbed with the double compound became very tender and soft, and of a red brown colour; that exposed to the triple compound hardened considerably, and became of a paler colour.

The triple compound, as I have mentioned before, has very little taste, and neither of the compounds seems to have any pernicious quality when received into the stomach. I fed a goldfinch with bread soaked in water, holding in solution the double compound for two days, and he drank water holding in solution the triple compound for three days, without apparently suffering any inconvenience.

Florence, March 23, 1814.

XXV. *Observations respecting the natural production of Saltpetre on the walls of subterraneous and other Buildings.* By John Kidd, *M. D. Professor of Chemistry at Oxford. Communicated by* William Hyde Wollaston, *M. D. Sec. R. S.*

Read June 16, 1814.

Although the following observations afford no positive evidence of the source of that saline efflorescence, which is so frequently seen on the walls of subterraneous and other buildings, and which, as consisting principally if not entirely of common nitre, long since gave rise to the name * by which that salt is most commonly known; yet as tending to throw some light on a very obscure part of natural history, they will not, perhaps, be unacceptable to this honourable and learned Society.

There can be no doubt that the production of saltpetre or nitre, in the situations above alluded to, had been observed long before there existed any general inducement to collect it from those sources; but after the invention and subsequent extensive employment of gunpowder, it became an object not only to search out every natural source of the principal ingredient of that important compound, but also to investigate the circumstances of its production; for the purpose either of accelerating the natural process, or of imitating it by artificial means.

* Saltpetre (Sal Petræ).

The usual and almost exclusive occurrence of saltpetre on walls constructed of limestone, and in situations exposed to animal and vegetable effluvia, in all probability led to the empirical practice of heaping together the mortar and refuse of old buildings with putrescent animal and vegetable matter; from a mixture of which kind, after exposure for a sufficient length of time to the action of the air, a quantity of nitre may usually be obtained by lixiviation: but it would be a question of mere curiosity, on this occasion at least, to investigate the origin of the practice. The intention of the present Paper is to state the result of a series of observations made during the last year, on the connexion that exists between the natural production of nitre and the state of the atmosphere. In detailing these observations, it will be convenient to give previously a description of the laboratory of the Ashmole Museum, in which building they were principally made: nor shall I be afraid of being thought too minute in this description, or in any other part of the following detail, by those at least who know the precision that is requisite in every induction, which like the present rests on phenomena of an obscure and equivocal nature.

The Ashmole Museum, which was built by Sir C. WREN in the reign of Charles the Second, is an insulated building, constructed entirely of calcareous freestone, and consisting of three stories. The lowermost of these stories was originally designed for, and has constantly been used as, a chemical laboratory.

The pavement of the laboratory, on its eastern, northern, and western sides, is about nine feet below the level of the street in which the Museum stands: on its southern side it is

on the same level with an area, about ten feet in breadth, which in part occupies the site of the ditch of the old town and insulates a quadrangular projecting part of the whole building of the Museum. The laboratory itself is a single room sixty feet in length from east to west, and twenty-five in breadth; having an arched stone ceiling, the centre of which is seventeen feet above the level of the pavement. The walls of this room, which are nearly three feet in thickness, are constructed of squared calcareous freestone, which I have reason to believe was dug from a quarry near Burford, and is technically called Windrush stone, from the river of the same name. There are four windows in the upper part of the north side of the laboratory, formed in the curve of the arched ceiling; the dimensions of each of which are five feet by four and a half.

There is no window either on the eastern or western side of the laboratory.

On the south side there are two windows, one at each extremity, looking into the area above described; and these windows are placed at the usual distance from the ground, that is, about three feet: and all that part of the south side intermediate to these two windows separates the laboratory from the quadrangular projecting part of the whole building of the Museum already mentioned.

The saline efflorescence takes place most copiously on the north wall, and it occurs on various parts of it from nearly the level of the pavement to within three or four feet of the centre of the arched ceiling. It takes place also, though not so abundantly, on the east and west walls; and also at the eastern and western extremity of the south wall; but it is

worth noticing, that I have never seen it on that part of the south wall which is common to the laboratory and the attached projecting building of the Museum. It is true that there are chimneys in this wall connected with fires that are lighted daily; but this circumstance does not seem sufficient to account for the absence of the nitre, because its formation takes place in another part of the laboratory equally near a chimney, and in which, from being inclosed, the temperature of the air is always considerably higher.

It is also worth noticing, that between the highest and lowest points of its appearance in every part of the building there are intercepted spaces entirely and always free from the least deposition.

I may here mention that the occasional formation of nitre is observable in many other buildings and parts of Oxford, besides the laboratory of the Ashmole Museum; as on the wall, called Long Wall, which bounds the park of Magdalene College to the west—on the exterior surface of the south wall of the Theatre—on the exterior surface of the three walls of the quadrangular projecting part of the Ashmole Museum—very abundantly on the inclined base of the windows of the Examination School, looking to the north—and also very abundantly on the west side of the wall, which separates the square of the Schools from the arched way leading from thence to the Theatre and Convocation House.

It has been observed repeatedly, that the presence of lime is necessary to the natural production of saltpetre; and in all the foregoing instances the stone on which the saline efflorescence takes place is the common limestone of Oxfordshire. I have only once observed its formation on the surface of a

brick wall: but in that instance the substance of those bricks on which the nitre appeared had crumbled away to some depth; and if this destruction of their texture be owing to the presence of an unusual proportion of lime in the clay of which they are made (a supposition not improbable, since many parts of the stratum of clay from which bricks in this neighbourhood are made do contain an unusual proportion of lime) the reason of the exception in the case of this brick wall will correspond with the truth of the general observation above stated.

The following circumstance is particularly deserving of notice. A part of the north wall of the laboratory, on which saltpetre usually effloresced, having been covered with wainscot some months since, and the wainscot having been painted with common white paint, I was surprised after a time in observing an efflorescence on particular parts of the paint, similar to what might have been expected on the wall itself. Where this efflorescence had taken place the paint was loosened from the wainscot, and might be readily peeled off in small flakes. The saline particles of which this efflorescence consisted I at first supposed to be nitrate of lead; but upon examination in various ways no trace of lead could be found in them, and they exhibited the principal characters of common nitre: they deflagrated, for instance, with charcoal, leaving a deliquescent alkaline residuum. Many weeks have elapsed since that saline efflorescence was brushed off, but I have not yet observed any renewal of it.

Though the production of saltpetre had been pointed out to me in the laboratory of the Ashmole Museum as long since as the year 1802, I was prevented by many circumstances from observing with any degree of regularity or precision

the phenomena of this natural process previously to the commencement of last year: about which time having carefully brushed away the whole of the saline efflorescence from a part of the north wall situated below the level of the street, and very imperfectly exposed to the light, I was surprised by the fact of its quick reappearance. It was brushed away at the end of January, but within three days it had again effloresced in sufficient quantity to present that appearance of hoar frost, or down, or mould, which is very characteristic of the manner in which naturally formed saltpetre is often accumulated; and which a person, utterly incapable of judging of the real nature of the substance, described at the time by the term "damp."

I was still more surprised, however, in observing after a few days, that the quantity of the saltpetre was apparently very much diminished; and that at the end of eight or ten days there was scarcely any appearance of it remaining: though there was no reason to suppose a particle had either accidentally fallen to the ground or been intentionally removed.

The same part of the wall was again brushed perfectly clean, and I continued to make daily observations in expectation of a renewal of the process; but no new collection was perceptible, excepting in the form of a very few detached and minute capillary crystals, till the 16th of March: on which day, having been absent during the whole of the 14th and 15th, I found a more abundant accumulation than in the preceding instances.

It was an obvious supposition that the reproduction of the nitre was connected with some change in the state of the atmosphere: and it is to be remarked, that its first mentioned

appearance took place in frosty weather, and that its reappearance about the 16th of March had been preceded by a frost of a few days; whereas during the intervals, in which it disappeared and was not again produced, the weather had been mild. The wall was again brushed quite clean on the 16th of March; between which day and the 4th of April, a considerable quantity had again collected on the same part; the process having been more rapid during the last four or five days, which were cold.

On April the 12th the front of this part of the wall was renewed by scraping, for the purpose of observing what would be the effect of a fresh surface, and before the 25th of April, nitre had formed on many parts of this new surface; but having first increased and then continued stationary for some days, it began to diminish about the beginning of May, the state of the air having been very moist during the two or three preceding days.

About the middle of May, 1813, I selected several distinct parts on which the spontaneous formation of nitre usually takes place, some within and some without the building of the Museum, and began to make daily observations on the phenomena of that process, which I noted down at the time: but as a register of this kind would be unnecessarily tedious in its recital, I shall beg leave to give the following general results of those observations; requesting it may be kept in mind that I do not presume to lay a greater stress on them than is proportional to the short period of time, and the circumscribed space in which they were made.

It appears then from the observations I have hitherto been able to make, that the spontaneous formation of saltpetre is,

generally speaking, much more extensive and rapid in winter than in summer, whether it take place on the interior or exterior of a building: thus during the period of the cold weather in January, 1814, it became visible not only on parts of the walls where I had never before observed it, but even on the pavement of the laboratory. The part of the pavement on which it appeared is adjacent to the north wall; but as the greater part of the pavement is covered by a flooring of wood, it is impossible to say whether or not it took place on other parts also. The stone forming the pavement, is the same kind of limestone as that of which the walls are built.

Again, whereas in summer its reproduction is most rapid and extensive in proportion to the degree of light present, the reverse of this (though not universally) takes place in winter. Wherever the saline efflorescence in question occurs, the surface of the stone becomes permanently discoloured, as if from the effect of damp; but this discoloration is merely superficial. If these discoloured parts be whitewashed, the process still goes on; and the whitewash is gradually detached in flakes: but it is difficult to ascertain whether the nitre is formed on the whitewash, or on the wall which it covers; though probably the latter.

When the spontaneous formation of nitre takes place slowly and in a sheltered situation, it is at first visible in the form of minute prismatic crystals, which usually project from the surface of the wall nearly at right angles; but sometimes they are scattered in different directions, lying upon its surface so lightly as scarcely to appear in contact with it. In general, however, the saline efflorescence makes its appearance in extremely minute capillary crystals, either accumulated

in groups, which resemble recently fallen flakes of snow, or investing the wall like a fine down.

During the severely cold weather of January, 1814, it appeared in some places in the form of minute dense grains closely aggregated; while in others it still continued to wear the appearance of down or wool: and the local circumstances most obviously connected with this difference in the manner of crystallization, were the presence of a greater degree of light, where the granular deposition took place, and a less degree of shelter from the influence of cold air.

In some instances the production of the saltpetre is accompanied with a disintegration of the substance of the stone on which it is formed: but this circumstance is only observable on stones of a loose texture.

The shortest interval I have observed between the time of its having been brushed away and its reappearance, is four hours: but it was then in full efflorescence, and would probably have been visible much earlier. The observation was made on November the 17th, about midnight: there had been snow in the middle of the day, and the night was frosty.

The spontaneous formation of nitre takes place indifferently on the surface of the stones composing a wall, and of the mortar by which those stones are cemented: and near the close of the late frost, I observed it for the first time on the surface of a partition, consisting entirely of laths plastered over with the mortar or stucco commonly used for that purpose.

It accumulates in greater quantity on some parts of a given surface than on others; and this difference in the degree of its accumulation, is probably connected with some slight difference in the texture or composition of the stone on which it is formed:

for I have repeatedly observed that after a careful removal of it, its reproduction takes place to the same proportional extent in the same parts: and this difference in the degree of the efflorescence often takes place not by a gradual transition, but as abruptly as if the line of separation had been marked by a graver; so that the part of the surface on one side of the line shall be almost totally devoid of any efflorescence, while on the other side it shall resemble the accumulation of hoar-frost on the small branches of a tree.

The saltpetre formed in summer scarcely appears to contain a particle of any calcareous salt: that formed in winter contains most evident traces of such a salt, though probably even in winter the amount of this is not much above one part in two hundred of the whole mass.

A frosty, clear, and dry state of the atmosphere, is particularly favourable to the natural production of nitre: but there seems to be a limit to its formation, on the same spot, even under the most favourable circumstances; the quantity of the nitre not continuing to increase after it has proceeded to a certain extent.

In a moist state of the atmosphere the formation either does not take place at all, or goes on slowly: and if that state of the atmosphere which is unfavourable to the production of nitre continue a sufficient length of time, the nitre already formed gradually disappears. At the commencement of these observations I attributed the occasional disappearance of the nitre to its mechanical removal from the wall, and supposed that it must have been by accident brushed off; but repeated observations convinced me this was not the case, its disappearance proceeding as gradually as its previous appearance;

besides which, had it fallen from the wall, I should most probably have found it on the pavement beneath, which never happened to me. It afterwards appeared probable, although I have never met with any condensed moisture on the surfaces submitted to the foregoing observations, that the aqueous vapour precipitated from the atmosphere in the state of it above alluded to, might dissolve minute particles of the nitre, and be absorbed with them into the substance of the wall; but on this supposition nitre ought to be found in lixiviating a portion of the stone taken near the surface. I have however made the experiment without detecting any nitre in the stone so taken. But, in opposition to the idea of the absorption of the nitre into the substance of the stone, I found the efflorescence disappear in more places than one during the severe frost of the present year (1814), at a time, when from the temperature of the stone, if not of the air also, the absorption here supposed could not have taken place; since, that temperature being below the freezing point, the aqueous particles would by congelation have been rendered incapable of dissolving the nitre.

It is deserving of notice, that such a spontaneous disappearance of the nitre, as has been just mentioned, took place antecedently to, and during the late heavy fall of snow on January 18 and 19.

Wishing to ascertain whether the free presence of atmospherical air be necessary in the natural process under consideration, I selected a part of the wall on which the formation of saltpetre usually takes place to a considerable extent; and insulated about a square foot of its surface which had been previously brushed quite clean. It was insulated by means of a plate of

glass, the edges of the frame of which were covered with a cement, so as to exclude any access of air between the glass and the wall. As the depth of the frame was not more than the third of an inch, the inclosed space contained but an inconsiderable quantity of atmospherical air. On the following day, October 29, I observed that a formation of nitre had taken place on the part of the wall within the glass, but that it was not so extensively diffused nor so abundantly deposited, as had been usual before the application of the glass; and the crystals, which were distinctly prismatic, were much more accurately defined, and larger than I had ever before observed them; in consequence, probably, of the tranquil state of the medium in which they were formed. On the day following, October 30, the quantity was increased; but it remained stationary from that time to November 12, when it began to diminish; and on November 16, there was no longer any appearance of it: nor did it reappear in the interval between November 16 and 29. In the mean time the efflorescence had not only taken place on the adjacent parts of the wall in the usual manner, but had been more than once brushed off and again deposited, while no increase had taken place in that originally formed within the glass.

On Nov. 29, the exterior surface of the glass was, for a purpose to be mentioned presently, covered over with whitewash; which was not removed till January 8, 1814. No observation could of course be made on the appearance of the surface within the glass during the interval that the glass itself was covered with whitewash; but no nitre was visible on the removal of the whitewash on January 8. It had, however, reappeared before the morning of January 10, partly in separate

capillary prismatic crystals which were much larger than on the former occasion, and reached from the surface of the wall to the inner surface of the glass; and partly in small flocculent tufts: the whole quantity of the nitre formed being much greater than, and the manner of its formation being very different from, that formation which took place within the glass on October 29 and 30. Still the whole quantity was not nearly equal to that which was usually formed on the same surface when exposed to the free action of the atmosphere. The increased quantity however formed in January, compared with that formed in October, corresponds with the observation that the natural production of nitre is much influenced by temperature: but it is worth mentioning that while the nitre formed within the glass in January occurred either in distinctly prismatic crystals or in flocculent tufts, that which was formed on the adjacent parts of the wall was of the granular character above described. (pag. 516, lin. 3.)

Having by the foregoing experiment satisfied myself with respect to the necessity of the free presence of atmospherical air for the full effect in the spontaneous production of nitre, I wished to ascertain the nature of the connexion between the nitre formed and the stone on which it is formed; and particularly whether carbonate of lime simply would be sufficient for its production. I therefore made a nearly liquid paste with distilled water and prepared chalk,* with which I covered the exterior surface of the glass that had been attached to the wall for the purpose already mentioned: but though nitre was

* As chalk sometimes contains magnesia, and almost always a small proportion of silex and alumine, I intend on a future occasion to repeat the experiment, using very finely pulverised crystallised calcareous spar, instead of chalk.

repeatedly formed in the neighbourhood of the glass, not a particle appeared on the whitewash during the space of nearly six weeks, from November 29 to January 8.

The foregoing statement contains the general results of all the observations I have hitherto been able to make on the present subject. I subjoin an imperfect analysis of the stone of which the laboratory of the Ashmole Museum is built, and of the saltpetre periodically formed on it; concluding with a few remarks on the local differences connected with a more or less extensive formation of that substance, and on its probable or rather possible source.

The stone, of which the laboratory is constructed, belongs to one of those alternating series of clay, and sand, and calcareous freestone, of which the varieties of Bath stone form nearly the lowermost strata; and the Headington stone, of which the greater part of Oxford is built, the uppermost. It is principally made up of the *debris* of small sea shells, but frequently contains organic vegetable remains, and always some proportion of sand and ochry clay.

Six portions of this stone taken from different parts of the laboratory, each weighing at least two hundred grains, were separately submitted to the action of a sufficient quantity of diluted muriatic acid. The undissolved residuum, which amounted in no instance to as much as four parts in a hundred, consisted of a yellowish brown ochry clay, mixed with a few particles of white sand, and small laminar fragments of a dirty white colour, very much resembling portions of the siliceous incrustation of the Geyser spring. Small flocculent

shreds slowly subsided in the muriatic solution, derived probably from the membranous part of the shelly matter of the limestone; for this insoluble residuum, when thrown into red-hot nitre, slightly deflagrated, and gave out an odour resembling burnt bones: and as this residuum had been collected without the use of a filter, the inflammable matter could not well have been derived from any other source than the stone itself.

The muriatic solution contained a small proportion of iron, and gave a very slight trace of magnesia. So that the composition of the stone may be thus stated.

Carbonate of lime - - - -	96
Oxide of iron, sand, ochry clay, and animal membrane	4
	100

In offering the following imperfect analysis of the saltpetre produced in the situations above alluded to, it is necessary to state, that the quantities on which I have hitherto had it in my power to operate have been very small.

Some preliminary experiments made on a solution of saltpetre which had been formed during the summer, gave the following results.

The solution contained no disengaged acid or alkali; and upon the addition of oxalate of ammonia afforded the slightest possible evidence of the presence of lime. It contained minute portions of sulphuric and muriatic acid. A portion of it being evaporated left a mass of crystals which, in their form, as in all their other characters, exhibited the properties of nitrate of potash.

As it is usually stated by writers who speak of the subject,

that nitrate of lime is a principal constituent part of saltpetre formed by a natural process, I was much surprised by the very slight trace of lime evident in the present instance; though I had felt assured previously that but little would be found in the natural saltpetre which I have had an opportunity of examining; having never observed in it any tendency towards deliquescence. Having had frequent occasion to suppose that carbonate of lime is much more readily soluble in water than is commonly believed, and having never been able to detach the saline efflorescence in question from the walls of the laboratory, &c. without admixture of particles of the limestone, or of the whitewash, amounting to at least seven or eight parts in a hundred, it struck me that these particles might be the source of the lime rendered evident by the addition of the oxalate of ammonia in the solution of the saltpetre. I therefore pulverised small portions of calcareous spar, of the common limestone of this country, and of whitewash; and having agitated accurately distilled water, at the common temperature, with each of these portions, I then filtered the water, and tested it with oxalate of ammonia. In each instance there was fully as copious a precipitate as when the oxalate of ammonia had been added to an equal quantity of the solution of saltpetre.

Similar preliminary experiments having been made on some saltpetre detached from the same part with that already submitted to examination, but formed during the winter instead of the summer, the same results were obtained with this single difference, that the precipitate obtained by the addition of oxalate of ammonia was much more copious: and I found this to be the case from whatever part of the laboratory or else-

where the saltpetre had been detached, provided it had been formed during the winter. I could not extend the experiment on the saltpetre formed during the summer, having only reserved a portion from one spot.

Judging from such experiments as I have made, it appears that the saltpetre formed in the situations described in this paper, consists of full 99 parts in 100 of nitrate of potash; with a very minute proportion of some calcareous salt, which is either not at all present in the saltpetre formed during summer, or is present in smaller quantity than in that formed during winter—though even in the latter instance it scarcely amounts to one part in 200.

The proportion of sulphuric and of muriatic acid, and the bases with which these acids are combined, I have not attempted to ascertain, on account of the minuteness of the proportion in which they evidently exist.

In considering the relative situations of the different parts of the laboratory, and the other buildings also, in which saltpetre is naturally formed, it is evident that the efflorescence takes place only where the exterior of the wall on which it is formed, is either exposed to the direct influence of the weather, or is in contact with the adjacent ground; not taking place at all in those instances in which the wall is neither exposed to the weather on either side, nor is in contact with the adjacent ground: from which it might be argued, that the effect depends upon the action of the external air, or of the moisture of the ground transmitted through the substance of the wall: but the result of that experiment in which, the action of the

air on the interior surface of the wall having been prevented by the intervention of a glass, the formation of nitre took place only to a very slight extent, is in a great measure at variance with such a supposition.

Again, though it is evident that the natural production of the saltpetre is closely connected with changes in the state of the atmosphere, those changes relating not only to its temperature but also to the degree of its moisture and barometrical pressure; yet a much longer series of observations, and these not partial like the present, but carried on in various parts of the world, is requisite, before even this part of the problem can be accurately solved. There still would remain a part of much more difficult solution, namely, the source of the metallic base of the alkali of the nitre.

With respect to this difficult question, if we compare the elements present in the composition of the saltpetre formed, with the elements of the substances present during its formation, it seems a reasonable conclusion that the potassium, or the metallic base of the potash of the saltpetre, is either a simple principle of some of the elements present, or that it results from the union of two or more of those principles, or of two or more of the elements themselves.*

The gross compounds present during the formation of saltpetre, in the instances mentioned in this paper, are atmospherical air and the limestone on which the saltpetre effloresces;

* The terms "element" and "principle" are here used in the same relation to each other as the early physical philosophers used the terms Στοιχεῖα and Ἀρχαί. They supposed that the former, though incapable of decomposition by common means, were not necessarily to be considered as absolutely simple substances: those absolutely simple substances they expressed by the term Ἀρχαί.

and all the component parts of these may be ultimately resolved into the following elements—oxygen, nitrogen, hydrogen, and carbon, together with iron and the metallic bases of some of the earths.

The component parts of nitrate of potash, of which the saltpetre under consideration almost entirely consists, are nitric acid, water, and potash; which may be resolved into the following elements—oxygen, nitrogen, hydrogen, and potassium: and all these elements are experimentally known to be present in the situations where saltpetre is formed, with the exception of potassium.

In the supposition then that the saltpetre is a product and not an educt of the above process, since no potash can be detected either in the air or limestone, the potassium must either be contained in a free state in the atmosphere or in the limestone, which from its remarkable attraction for oxygen is inadmissible: or it must be a component principle of some one of the elements present; or lastly, it must be itself a compound of two or more of the principles of those elements, or of two or more of the elements themselves.

But I am aware that the saltpetre may be considered as an educt of the process, and that it may possibly exist already formed in the atmosphere in a state of minute division: yet, when we consider the comparatively fixed nature of that salt, and that no experiments have yet detected its presence in atmospherical air, or in the moisture precipitated from atmospherical air; and lastly, when we reflect on the probability that the metals, of which potassium is one, are compound bodies, the former supposition seems upon the whole preferable to the latter.

February, 1814.

XXVI. *On the Nature of the Salts termed triple Prussiates, and on Acids formed by the union of certain Bodies with the Elements of the Prussic Acid.* By Robert Porrett, *jun. Esq.* *Communicated by* William Hyde Wollaston, *M. D. Sec. R. S.*

Read June 30, 1814.

Since the admirable experiments of Scheele on the Prussic acid, chemists have distinguished between the salts formed by the direct combination of this acid with alkaline or other bases, and those obtained either by adding to the salts just mentioned a certain portion of a metallic oxide, or by decomposing by the alkalies a prussiate of a metallic oxide. Thus for instance, the salt formed by the union of the Prussic acid with potash has been called the simple prussiate of potash, while that obtained by boiling a solution of potash on Prussian blue has been called the triple prussiate of potash and iron, it having been proved that the black oxide of iron enters into its composition. The differences between these salts are very remarkable, the simple prussiate is always alkaline, is soluble in alcohol, is incapable of forming Prussian blue when added to the salts of per-oxide of iron, gives a yellow precipitate with salts of per-oxide of copper, is decomposed at the temperature of boiling water, and has its acid displaced by the carbonic acid. On the contrary, the triple prussiate is neutral, is insoluble in alcohol, forms Prussian blue with the salts of per-oxide of iron, gives a reddish brown precipitate with salts of per-oxide of copper,

is not decomposed at the boiling temperature, nor in the least altered by carbonic acid. To the black oxide of iron, therefore, existing in this salt, chemists have attributed its superior permanency to the simple prussiate, and also its conversion to the neutral state. The latter effect of the oxide of iron is without a parallel in chemistry, no other alkaline salt is known which can be rendered neutral by the mere addition of black oxide of iron, which in all other instances will neutralize acids instead of alkalies. This however is not the only anomaly which the triple prussiates present, the following being also very remarkable.

The black oxide of iron contained in them cannot be thrown down by the addition of any of the hydrosulphurets, succinates, gallates, or any other tests for iron.

This oxide always accompanies the Prussic acid, when the latter is transferred by stronger affinities from one base to another.

For most of the facts just related, I am indebted to the excellent paper of PROUST, entitled "Facts towards a History of the Prussiates." It appeared to me to be necessary to restate these facts in order to render clear the objects of this communication, which are to reconcile with the general properties of other saline bodies the anomalies which the triple prussiates present; to simplify the received opinion respecting their nature, and to make known some new acids which I have discovered.

In endeavouring to accomplish these objects, I shall first state my opinion of the nature of the triple prussiates as they are called, and afterwards support it by experiments.

I consider the salts termed triple prussiates as binary com-

pounds of an acid with a single base; as salts which do not contain any Prussic acid, nor any oxide of iron *as a base*, although both these substances may be obtained from them by a decomposition of their acid.

The first experiment which I shall adduce in support of the above opinion is one with the Voltaic battery; it appeared to me that this instrument would shew whether the oxide of iron in the triple prussiates existed in them as a base, or as an element of a peculiar acid by attracting it to the negative pole in the former, and to the positive pole in the latter case. I therefore exposed a solution of triple prussiate of soda to the agency of a small battery of fifty pair of double plates of one inch and a quarter square, kept in action for twenty hours, the solution was connected by platina wire with the negative pole, and by filaments of cotton with distilled water which communicated by platina wire with the positive pole. Thus circumstanced the triple prussiate of soda was decomposed, its acid (consisting of the elements of prussic acid and black oxide of iron) was carried over to the positive pole; here, meeting with oxygen from the decomposing water, it underwent a farther change by which it was converted into Prussic acid which was partly volatilized, and into blue triple prussiate of iron which formed there in abundance; the liquid at the negative pole after this process contained only soda with a trace of undecomposed triple prussiate. In this experiment I consider the circumstance of the black oxide of iron being carried over to the positive pole as a proof that it went there as an element of an acid, for as a base it must have remained at the negative pole.

I have repeated the above experiment with only the variation

of employing conducting wires of copper instead of platina, and found that the only difference in the result was, that the reddish brown triple prussiate of copper and iron was formed instead of the blue triple prussiate of iron, which effect was owing to the oxide of copper formed at the positive pole combining with the peculiar acid of the triple prussiates.

The next experiment which I shall relate is still more conclusive. I dissolved the triple prussiate of barytes in cold water, of which it required 1800 times its weight, I divided this equally, and by an experiment made purposely with one portion, ascertained the exact quantity of diluted sulphuric acid necessary to throw down the barytes from the other; this quantity was equal to 2,53 grains of real sulphuric acid for every ten grains of the salt employed; then filling a bottle with the second portion so as only to leave room for the acid to be added, I poured into it the quantity of acid previously ascertained, closed the bottle immediately, and left the sulphate of barytes to subside to the bottom; when this had taken place, the clear supernatant fluid was examined, and found to be the pure acid of the triple prussiates. It has the following characters:

It is of a pale lemon yellow colour; it has no smell; it is decomposed by a gentle heat, or by exposure to a strong light; prussic acid is then formed, and white triple prussiate of iron, which quickly absorbing oxygen from the air, changes to blue triple prussiate. It forms directly with alkalies, earths, or oxides, the salts termed triple prussiates. It displaces the acetic acid from all its combinations in the cold, and forms triple prussiates with the bases before united to that acid. It is capable of expelling all other acids from their soluble

combinations, when it can form with their bases compounds that are insoluble in acids.

The decomposition of the abovementioned peculiar acid by heat, shews the reason why the acid of the triple prussiates can never be obtained from them by distilling these salts with a stronger acid, and why in these cases, Prussic acid and triple prussiate of iron are always obtained.

By proving that the oxide of iron contained in the triple prussiates is an elementary part of an acid *sui generis*, the several anomalies mentioned in the beginning of this Paper are easily explained, for instance, its occasioning the simple prussiates to pass from an alkaline to a neutral state, is owing to its constituting with the other elements present, a new acid having much stronger acid properties than the Prussic. Its not being affected by the usual tests for iron is owing to its existing as an element of an acid, in which state it is no more to be expected that it should be discovered by the ordinary reagents, than that the sulphur in sulphuric acid should blacken metallic solutions. Finally, its accompanying the elements of the Prussic acid in all cases of transfer from one base to another, is owing to its forming with these elements the acid of the triple prussiates, which acid, when transferred by stronger affinities, carries with it of course all its component parts. I have discovered that besides the black oxide of iron there are other substances which are capable of forming, with the elements of Prussic acid, peculiar acids having properties widely different from each other.

Sulphur is one of these substances.

Several of the metallic oxides, I have reason to think, are

also of this kind, but I have not had leisure to investigate particularly the acids that may thus be formed.

That with sulphur however has engaged much of my attention, as may be seen by reference to the continuation of this Paper.

The true nature of the triple prussiates being, as I hope, now developed, an alteration in their nomenclature appears indispensible, as those salts can no longer with propriety be called triple salts which have but one acid and one base, nor can they be called prussiates which do not contain any prussic acid. Besides, it seems full time that the term prussiates should be restricted to the simple compounds of Prussic acid with bases, and that salts so very different from them, as are the triple prussiates, should have a name so dissimilar as to prevent for the future any farther confusion between them; I propose therefore for the acid composed of the elements of the Prussic acid united with oxide of iron, the name of ferruretted chyazic acid, for that composed of the same elements with sulphur the name of sulphuretted chyazic acid, for those consisting still of the same elements with metallic oxides analogous names, as argenturetted chyazic acid, &c. the word chyazic being composed of the first letters of the acidifiable elements of the acid, viz. carbon, hydrogen, and azote, to which is added the termination *ic*.

I shall now relate some analyses that I have made of the triple prussiates, or as I shall henceforth call them, the ferruretted chyazates.

Analysis of ferruretted Chyazate of Potash. (Triple Prussiate of Potash.)

I cannot find that this salt has ever been completely analysed: the only person who to my knowledge has even partially effected it is PROUST, who confined himself to ascertaining the quantities of water and of black oxide of iron which it contains, and which he states to be in the 100 parts,

Water - 10
Black oxide of iron 17

I have made similar experiments to those from which he drew the above conclusions, and added others of my own; the following are the particulars of these experiments with the results which I have obtained.

A. 100 grains of the crystallized salt, finely pulverized, were kept for six hours at the temperature of 212°, by this treatment they were reduced in weight thirteen grains; they were afterwards heated for one hour at the temperature of 40°, but they did not suffer any further loss.

B. 100 grains of the same salt were heated with weak sulphuric acid until all the Prussic acid that could be expelled as gas was separated, the rest of the Prussic acid combined during the process with the oxide of iron contained in the salt, and formed ferruretted chyazate of protoxide of iron (white prussiate of iron), which I afterwards changed into ferruretted chyazate of per-oxide of iron (blue prussiate of iron) by the addition of a little weak nitric acid; it was then thoroughly washed and dried at 212°, it weighed after this thirty-five grains. These thirty-five grains, as may be seen by the analysis which I have made of the ferruretted chyazate of

per-oxide of iron by its combustion, which will be related presently, contain 6.76 grains of black oxide of iron and 11.71 grains of red oxide of iron, but as the whole of the iron in the ferruretted chyazate of potash is in the state of black oxide, these 11.71 grains of red oxide must be reckoned as 10.5 grains of black oxide, which added to the 6.76 grains gives 17.26 as the quantity of black oxide of iron contained in 100 grains of the ferruretted chyazate of potash.

C. 100 grains of crystallized ferruretted chyazate of potash dissolved in water and poured into a solution of green sulphate of iron gave a precipitate which, treated exactly as the last, was ferruretted chyazate of per-oxide of iron and weighed 89.3 grains, the elements of the Prussic acid contained in which weigh 30.4 grains, as may be seen in the analysis before referred to.

D. 100 grains of the same crystallized salt, decomposed by sulphuric acid, gave a quantity of sulphate of potash which, after fusion in a platinum crucible by a strong red heat, weighed 78.5 grains: now as 100 grains of sulphate of potash in this state contain, according to Dr. THOMSON (Annals of Philosophy for Oct. 1813), 50.1 grains of potash, therefore these 78.5 grains must have contained 39.33 of that alkali.

Hence, neglecting minute fractions, the crystallized ferruretted chyazate of potash is composed of

Grains.			Grains.
	Water A		13.00
17.26	black oxide of iron B	ferruretted chyazic acid	47.66
30.40	elements of Prussic acid C		
	Potash D		39.34
			100.00

Analysis of ferruretted Chyazate of Barytes. (Triple Prussiate of Barytes.)

A. 100 grains of this salt exposed to the temperature of 340° until they no longer lost weight, parted with 16.59 grains of water.

B. 10 grains of the salt were added to about eight ounces of water acidulated with muriatic acid to increase its solvent power on the salt; when the whole was dissolved, a solution of supersulphate of potash was poured into it as long as it occasioned any precipitate, this precipitate after having been well washed, dried, and ignited in a platinum crucible, weighed 7.44 grains, which, according to BERZELIUS's analysis of sulphate of barytes, contain 4.91 grains of barytes.

C. As this salt consists only of ferruretted chyazic acid with barytes and water, the quantity of the two last being ascertained leaves for the other in the 100 grains, 34.31 grains.

100 grains of ferruretted chyazate of barytes is therefore composed of

Water A	- - - -	16.59
12.42 Black oxide of iron 21.89 Elements of Prussic acid	} ferruretted chyazic acid C -	} 34.31
Barytes B	- - -	49.10
		100.00

Analysis of ferruretted Chyazate of Per-oxide of Iron. (Blue Prussiate of Iron.)

A. I found that in order to form 100 grains of ferruretted chyazate of per-oxide of iron, it was necessary to add 112

grains of ferruretted chyazate of potash to a solution of iron; now by my analysis of this salt, it appears that the quantity of acid in these 112 grains is 53.38 grains, the whole of which acid enters into the composition of the 100 grains of blue precipitate.

B. 124 grains of crystallized green sulphate of iron being dissolved in water, there was required to precipitate all the oxide of iron from it, a solution containing 112 grains of crystallized ferruretted chyazate of potash; now 124 grains of crystallized green sulphate contain, according to BERZELIUS, 31.44 grains of protoxide of iron, but as this iron is in the state of per-oxide in Prussian blue, it must be reckoned as 35 grains.

C. PROUST has stated that the quantity of per-oxide of iron procured by calcining 100 grains of Prussian blue is 55 grains, I have repeated this experiment and obtained the same result; of these 55 grains, 21.53 grains must have been derived from the 53.38 grains of acid in the Prussian blue (see Exp. A), in which they existed as black oxide, weighing only 19.38 grains.

D. I attempted to obtain by distillation the water that is contained in Prussian blue, that had been dried as much as possible at 212° by heating 100 grains of it at the temperature of melting tin in a small retort adapted to a receiver containing fused muriate of lime, the retort and the receiver with their contents being separately weighed, both before and after the experiment; but I found that only an approximation to the truth could be gained by this method, as the water that came over held prussiate of ammonia in solution, and as some of the same prussiate escaped as gas: after the experiment

I found that the contents of the retort had lost in weight 19.5 grains, and that those of the receiver had gained 10.5, the mean between these two is 15 grains, which does not differ very materially from the quantity deduced from ascertaining the weight of all the other constituents of Prussian blue, consequently this latter method may be adopted without the hazard of any material error.

Ferruretted chyazate of per-oxide of iron according to these experiments is composed of

19.33 Black oxide of iron 34.05 Elements of Prussic acid	Ferruretted chyazic acid A -	53.38
	Per-oxide of iron B	35.00
	Water D	11.62
		100.00

or if its composition be calculated from the quantity of per-oxide of iron afforded by its combustion, jointly with the quantity of ferruretted chyazate of potash required to form it, then it will consist of

Ferruretted chyazic acid A -	53.38
Per-oxide of iron C -	33.47
Water D - - -	13.15
	100.00

These analyses by different methods agree together as nearly as could well be expected, the mean of the two is given below, which cannot, I think, be far from the truth.

Ferruretted chyazic acid	53.380
Per-oxide of iron	34.235
Water	12.385
	100.000

Before I dismiss the subject of the ferruretted chyazates, I beg to make some remarks on the opinion generally entertained by the most eminent chemists, of the inadequacy of these salts as tests for iron capable of affording correct information of the *quantity* of oxide of iron that may be present in a solution: in doing which I hope to shew that this opinion is erroneous.

Remarks on the objections made to the use of the ferruretted Chyazate of Potash as a test for the quantity of Iron in Solutions.

These objections I find very well stated by Dr. MARCET, in his Chemical Account of an aluminous Chalybeate Spring in the Isle of Wight, as follows:

"The prussiate of potash, however well prepared, is quite "inappropriate when the object is to ascertain the *quantity* of "iron in solution, its utility being confined to indicating its "presence, it being liable to the objections of precipitating "some earthy substances, particularly alumine, when the mix-"ture is heated, of being itself decomposed by heat, or of "not separating the iron so as to be retained on the filter or "procured by subsidence, if heat is not applied, or if the iron "is not highly oxidated."

Notwithstanding the authority which an opinion of Dr. MARCET's on this subject justly carries with it, I must take the liberty of asserting, that although I agree with him in the

experimental facts cited as the causes for it, yet that I draw a different conclusion from his, because, instructed by my experiments on the nature of the triple prussiates, I find that all those inconvenient effects are easily prevented from ensuing; and I therefore conclude that the ferruretted chyazate of potash is in general the most appropriate test of the quantity of iron in solution of any employed in chemistry; the use of it for this purpose only requiring attention to the following facts.

1st. That if the ferruretted chyazate of potash after being dissolved in water gives immediately a blue precipitate by the addition of muriatic acid, it is not pure, and will give a fallacious result.

2dly. That if this salt, however pure, be added *in excess* to a solution of iron *containing an excess of acid*, and then heated, the Prussian blue thrown down will weigh more than it ought, because some is furnished by the decomposition of the ferruginous acid of that part of the salt which has been added in excess.

3dly. That Prussian blue, even after it has been formed, is materially acted upon by a mixture of nitric and muriatic acids, and in some degree by the muriatic acid alone at a boiling heat.

4thly. That Prussian blue, when precipitated, often carries with it sulphate of potash derived from the liquid from which it is thrown down; and that this sulphate adheres to it so obstinately that several washings with water acidulated with sulphuric acid, are necessary to detach it.

5thly. That if the solution to which the test is applied contains besides iron, barytes, alumine, oxide of copper, or any

other substance which the ferruretted chyazate is known to precipitate, then it is necessary to separate such substance by the usual means, before the application of this test for iron.

With the knowledge of these facts, therefore, it is easy always to obtain correct results, which will best appear by assuming a case particularly unfavourable for the purpose, and shewing how the difficulties of that case may be obviated.

Let it therefore be supposed that the analyst has a solution having a considerable excess of nitromuriatic acid and containing oxide of iron, barytes, alumine, magnesia, and oxide of copper, and that he wishes to precipitate the iron by the ferruretted chyazate of potash. If he first throw down the barytes by a sulphate, then the other earths and oxides by ammonia, adding an excess of ammonia to redissolve the oxide of copper, if he then separate the alumine by digestion in caustic potash, and redissolve the oxide of iron and magnesia in an acid, which for the sake of increasing the difficulty may again be supposed to be the nitromuriatic; I say then, that by neutralizing any very great excess of acid, and adding some acetite of potash in order that the unavoidable excess of acid may be the acetic instead of the nitromuriatic; then applying the ferruretted chyazate of potash, taking care that there be but little, if any, excess of it added; he may safely boil the liquid and readily collect by the filter, or by subsidence, the ferruretted chyazate of per-oxide of iron, which when properly washed, dried, and weighed, will indicate very accurately the quantity of iron that was in the solution, by reckoning that quantity as 34.235 of per-oxide, for every 100 parts of Prussian blue obtained.

The succinates and benzoates which have been proposed as superior tests for determining the quantity of iron, to the ferruretted chyazates, I consider to be in general very inferior to them, because they do not precipitate the iron, when it is in the state of protoxide, which the latter do, and because they precipitate alumine more effectually than those salts.

On sulphuretted Chyazic Acid.

I discovered this acid in the latter part of the year 1808, whilst engaged in an experiment the object of which was to form prussiate of potash by the mutual action of Prussian blue and sulphuret of potash when boiled together; I found after this boiling had been continued for a considerable time with fresh additions of Prussian blue, until the alkaline liquid was completely neutralized, that instead of prussiate of potash which I had expected to obtain, a new principle had been formed which had properties totally different from the Prussic acid, or from any other known chemical substance. I made experiments in order to separate this principle from the various products which were mingled with it in the liquid which I obtained, and having succeeded in this, I found that in its pure state it had acid properties, and was capable of being converted by oxidizing agents into Prussic acid; this circumstance considered in conjunction with its being formed by a deoxidating process on a salt of iron containing the Prussic acid, led me to conclude, that this new acid differed from the Prussic acid only in containing less oxygen, I therefore named it Prussous Acid, and in a paper which I presented to the Society of Arts in April 1809, detailed the circumstances of its formation, the mode of obtaining it free from foreign bodies,

and its effects on many of the metallic solutions. Since the experiments were performed which I then communicated, I have made several others which have added considerably to my knowledge of the nature and properties of this peculiar acid, and proved to me that my first opinion of its nature was incorrect. I shall endeavour in this paper to give as concise an account of these experiments as I am able, consistently with making myself understood, notwithstanding which I fear that from their number it will not be in my power to compress them within moderate limits; in attempting this, however, I shall arrange them under the following heads:

1st. Experiments on this acid of a synthetical nature.

2dly. Experiments on the same of an analytical nature.

3dly. Experiments on the salts formed by this acid.

Before I detail the experiments of the first and second class which relate to the nature of this acid, I beg to observe that in describing them I shall designate this acid provisionally by the name of the red tinging acid, as neither the name of Prussous acid, which I first gave to it, nor that of sulphuretted chyazic acid, which I now adopt, could in this place be used with propriety, as they each imply different compositions of this acid, the true nature of which, until these experiments are related, must be presumed to be doubtful.

Synthetical Experiments on the Red Tinging Acid.

Under this head I shall include not only the different processes by which I have actually formed this acid, but those also by which I have attempted to form it without success, as both kinds of experiments throw light upon its nature; I shall

add also the account of the processes by which I have obtained it pure from the liquid in which it is formed.

Processes by which I have formed the Red Tinging Acid.

A. The first is the one already mentioned, which consists in boiling, for a considerable length of time, three or four parts of Prussian blue in powder added at as many intervals, with one part of sulphuret of potash, which sulphuret by the action of the hot water upon it becomes an hydroguretted sulphuret. The products of this process are black sulphuret of iron, and a colourless neutral liquid containing a considerable quantity of the red tinging acid combined with potash, much sulphurous acid in union with sulphur and potash, forming a sulphuretted sulphite of potash, and some sulphate of potash.

B. The second process is analogous to the preceding, it consists in decomposing prussiate of mercury by hydroguretted sulphuret of potash, this decomposition, owing to the solubility of this prussiate, is instantaneous, while the former only takes place after a tedious ebullition; the products of it do not differ in any respect from the former, with the exception, that instead of the black sulphuret of iron, there is formed the black sulphuret of mercury.

C. The third process consists in heating to redness for some hours a mixture of animal charcoal and sulphuret of potash, and, when the mass is cold, lixiviating it with water. This process, which is by far the most economical of any, gives the same results as the former, with the exception of there being no metallic sulphurets, and that the liquid generally contains much undecomposed hydroguretted sulphuret of potash.

D. The fourth process is only a variation of the third, by substituting for the sulphuret of potash, the sulphate of that alkali, which sulphate is converted to sulphuret during the process by part of the animal coal; I performed this process but once, but it did not appear to me to give so large a quantity of the red tinging acid as the last.

E. The fifth process consists in forming the simple alkaline prussiate of potash by passing Prussic acid gas through a solution of that alkali, and afterwards boiling this liquid alkaline prussiate with a little sulphur, the whole of the Prussic acid in this salt is thus changed into red tinging acid.

F. The sixth process is very analogous to the last, it consists in adding to the simple prussiate of ammonia, hydroguretted sulphuret of potash, by which the simple prussiate of ammonia is immediately changed into the compound of ammonia with the red tinging acid. The liquid after the process always contains sulphuretted hydrogen, even when the hydroguretted sulphuret has been added in a quantity insufficient to convert the whole of the Prussic acid into red tinging acid.

These are the various processes by which I have formed the red tinging acid, those by which I could *not* form it are now to be mentioned.

G. *Experiment* 1. Ammoniacal gas was passed through a red hot earthen tube containing charcoal, by this process as is well known Prussic acid is abundantly formed, but my object in performing it, was to ascertain whether any of the red tinging acid was at the same time produced; not the slightest trace of it, however, could be detected in any of the products.

H. *Experiment* 2. Phosphuret of lime was thrown into the simple prussiate of ammonia, phosphuretted hydrogen was as usual produced, but no red tinging acid was formed.

I. *Experiment* 3. Phosphuret of lime thrown into prussiate of mercury threw down the mercury as a black powder, the liquid after this contained prussiate of lime, but none of the red tinging acid.

K. *Experiment* 4. Phosphuretted hydrogen gas was passed through prussiate of mercury, the mercury was thrown down, and Prussic acid set free.

L. *Experiment* 5. Arseniuretted hydrogen gas acted in a similar manner.

M. *Experiment* 6. Sulphuretted hydrogen gas passed through the same salt occasioned similar phenomena, black sulphuret of mercury being formed and Prussic acid disengaged.

N. *Experiment* 7. The same gas passed through the simple prussiate of ammonia did not alter that prussiate.

O. *Experiment* 8. 0.5 of a cubic inch of the same gas, at mean temp. and pressure being passed up into a tube over mercury, and an equal volume of Prussic acid gas being added, the volume of the two gases was not altered by the mixture, nor did any combination take place.

P. *Experiment* 9. The preceding experiment was repeated with the addition of passing 150 electric discharges through the two gases, but no combination ensued.

Q. *Experiment* 10. A repetition of Experiment 8, with the addition of 0.5 of a cubic inch of chlorine gas, the volume of the mixed gases was diminished to one-third of their former bulk, and sulphur was deposited on the inside of the tube, but the Prussic acid gas was unaltered.

R. *Experiment* 11. 0.5 of a cubic inch of sulphuretted hydrogen gas, the same volume of Prussic acid gas, and 0.25 of a cubic inch of oxygen gas, were attempted to be exploded by the electric spark, but they did not form an explosive mixture in these proportions, I therefore added 0.125 of a cubic inch more of oxygen, and again passed the electric spark through them, an explosion now took place attended with a diminution of volume amounting to 0.5 of a cubic inch; no red tinging acid was contained in the product of this experiment.

S. *Experiments* 12, and 13. Hydrosulphuret of ammonia added to simple prussiate of ammonia, and to prussiate of mercury, did not occasion the formation of any of the red tinging acid.

T. *Experiments* 14, 15, and 16. Sulphuret of carbon added to the two prussiates just mentioned, and also to liquid Prussic acid, was equally ineffectual as the hydrosulphuret.

U. *Experiments* 17, 18, 19, and 20. Sulphur was added to the same three liquids as the sulphuret of carbon, some sulphur was also kept in fusion while Prussic acid gas was passed through it, in neither of these cases was any chemical change effected.

V. *Experiments* 21, 22, and 23. Hydroguretted sulphur was added to the same three liquids with no better success.

W. *Experiments* 24, 25, and 26. Sulphur added to a strong solution of pure potash in the cold, so that no hydroguretted sulphur could be formed, was very sparingly dissolved in the alkali, the solution, or rather mixture of the two, was poured into each of the three liquids before mentioned, it did not alter the acid contained in them.

X. *Experiments* 27, 28, and 29. Sulphuretted sulphite of potash, prepared by adding sulphuret of potash to strong sulphurous acid, was employed in the same manner, but like all the preceding substances, it was found to be incapable of forming an atom of the red tinging acid.

The preceding series of experiments comprises all that I have made with the view of forming the red tinging acid; they appear to prove,

1st. That this acid cannot be formed without sulphur. GH IKL.

2d. That it cannot be formed with sulphur, unless that substance be in the state of hydroguretted sulphur. MNOPQRS TUWX.

3d. That it cannot be formed with hydroguretted sulphur, without the presence also of an alkali. V.

It will be seen by reference to the successful experiments, that in them these essential conditions were fulfilled; hence the indispensible agents in producing that peculiar change on the Prussic acid which constitutes it the red tinging acid, may be considered as determined; the nature of the change which they effect will be best understood, after relating my analytical experiments on the red tinging acid, previous to doing which, I have to describe my process for obtaining the acid in a pure state from the liquid in which it is formed; this process is the same in whatever mode the liquid has been obtained, it is the following:

Let the clear liquor be brought to a decidedly acid state by the addition of sulphuric acid, then keep it for a short time at nearly the boiling point, when cold, throw into it a little native black oxide of manganese finely pulverised, which will turn it of a beautiful crimson colour, filter this crimson fluid, and add to

it a solution containing sulphate of copper and green sulphate of iron, in the proportions of two of the former salt to three of the latter, until the crimson colour disappears, when a copious white precipitate will be formed, which is an insoluble salt of copper composed of the protoxide of that metal combined with the red tinging acid. This salt must then be boiled with a solution of potash which takes from it its acid, leaving the oxide of copper as a bright yellow powder. The acid thus transferred to the potash may be obtained in a free state by mixing this combination with sulphuric acid, and distilling it. The red tinging acid comes over, as a liquid strongest towards the end of the process, it is not however yet quite pure, as it will be found to contain a little sulphur and sulphuric acid, the best mode of separating both of which, is to add carbonate of barytes until the acid is saturated with it, and afterwards to throw down the barytes by the careful addition of sulphuric acid. The red tinging acid will thus be obtained in a state of perfect purity. In this state it is generally colourless, but sometimes acquires a pinkish hue, which I believe to be owing to a commencement of oxidation. In the most concentrated form in which I have obtained it, it had the specific gravity of 1.022, and a smell as pungent, and a little resembling that of strong acetic acid; at the boiling temperature it had a slight solvent action on sulphur, when the latter had been presented to it in a very divided state, but most of the sulphur thus dissolved separated again on cooling, the rest which it still retained after cooling, and which is foreign to its composition, may be detected by nitrate of silver, or by nitrate of protoxide of mercury, which form with the red tinging acid in the pure state, precipitates which are white and continue so, but with the acid holding sulphur in

solution, these precipitates have rather a dark colour. I proceed now to relate my

Analytical Experiments on the Red Tinging Acid.

A. I find that whenever this acid is distilled, a portion of sulphur remains behind in the retort, and the acid that passes over into the receiver contains a little in solution, at the same time a portion of the acid disappears, so that it would be possible, by repeated distillations, to destroy the whole of it. The nature of the new product produced by this repeated distillation I have not investigated any farther than to ascertain that it did not contain any Prussic acid. The sulphur that was separated was not in the hydroguretted, but in the common state.

B. Some of the red tinging acid was added to nitrate of lead, and an excess of nitric acid was poured into it. No precipitate was formed in the cold, but when heat was applied, a considerable action took place, attended with the escape of nitrous gas, and a copious white precipitate appeared, which, upon examination, proved to be sulphate of lead.

C. Five grains of the white salt of copper before mentioned, prepared with the most scrupulous care, and which I knew by an analysis (to be presently mentioned) contained 1.84 grain of this acid in a dry state, were totally decomposed by nitric acid in which muriate of barytes had been previously dissolved: they afforded by this treatment a white powder, weighing after ignition 15.82 grains, which by the subsequent action of nitric acid were reduced to 8.86 grains, and were sulphate of barytes, which is equivalent to 1.20 grain of sulphur.

D. Some of the red tinging acid was heated upon native

carbonate of copper or malachite in powder: during the operation there was a constant escape of carbonic acid from the malachite, and of Prussic acid into which one part of the red tinging acid was converted, whilst another part formed the white powder containing the protoxide of copper: now as all the copper in this ore was in the state of per-oxide, it follows that part of this oxide must have relinquished oxygen to some principle contained in the red tinging acid; the principle thus oxidated was found to be sulphur, for after the experiment on testing the liquid for sulphuric acid, the presence of that acid was most unequivocally demonstrated.

E. Some of the red tinging acid was mixed with diluted muriatic acid and hyperoxymuriate of potash; a gentle heat was then applied until chemical action took place, when the heat was withdrawn and the action moderated by cooling; by this process sulphuric acid was formed in the liquid, and Prussic acid also, part of which escaped as gas, and the rest was readily discovered in the liquid by neutralizing it and adding a solution of iron, with which it formed Prussian blue.

F. The red tinging acid was electrified negatively in the Voltaic circuit, and connected by cotton threads with distilled water electrified positively. The result of this experiment was, that the acid was carried over to the positive side without decomposition, or any separation of sulphur.

G. The above experiment was repeated, adding to the acid in the negative cup potash, and pouring into it mercury, into which the copper wire dipped that communicated with the battery. The result was the same as in the last experiment; the acid was carried over to the positive pole unaltered, and neither the potash, the mercury, nor the copper wire had

combined with any sulphur. That sulphur existed in this acid, both before and after this experiment, was however proved, by heating it in a mixture of nitric acid with muriate of barytes, by which means sulphate of barytes was quickly formed and precipitated.

From these analytical experiments on the red tinging acid, it appears to me that the following inferences may be drawn,

1st. That this acid contains sulphur. ABCDEG.

2dly. That this sulphur constitutes two-thirds of the weight of the dry acid. C.

3dly. That it exists in it, not as a foreign body in solution, but as an element. FG.

4thly. That when the sulphur is acidified, the other constituents of this acid unite in the form of Prussic acid, and that therefore they consist of the same elements as that acid. DE.

The results of these analytical experiments shew clearly why, in the synthetical ones, the presence of sulphur for forming it from Prussic acid was indispensible; they do not however so obviously shew why, besides the sulphur, the presence of both hydrogen and an alkali are requisite; probably the use of the hydrogen is to bring a sufficient quantity of sulphur into a state in which it is soluble in the alkali, while that of the alkali, by virtue of the mysterious power often termed disposing affinity, favours the formation of the stronger or red tinging acid, in consequence of its having a greater attraction for it than for the weaker Prussic.

I shall now drop the term red tinging acid, used merely in relating these experiments to avoid anticipating its nature, and having, as I think, proved that this acid is composed of sulphur and of the same elements as the Prussic acid, and that

consequently it belongs to the same class of acids as does the ferruretted chyazic acid, I shall call it henceforth the sulphuretted chyazic acid, and conclude this communication with giving some description of its salts, with my analyses of two of the most important of them, and by which the composition of all its other salts may be known with comparatively little trouble.

Sulphuretted Chyazates.

Of potash, a deliquescent salt soluble in alcohol.

Of soda, ditto, crystallizes in rhombs.

Of lime, ditto, soluble in alcohol, from which it may be obtained as a mass of needle form crystals.

Of ammonia, ditto, not crystallizable.

Of magnesia, ditto, when dried has a micaceous appearance.

Of alumina, crystallizes in octohedra which do not deliquesce.

Of barytes, a deliquescent salt, crystallizing in long slender prisms of a brilliant white colour.

Of strontia, a deliquescent salt crystallizing in long slender prisms in groups radiating like zeolite.

Of oxide of silver, a white insoluble powder.

Of protoxide of mercury, a white insoluble powder.

Of potash and prussiate of mercury, this salt is remarkable for its brilliant silvery lustre, and for its very compound nature, as all the four principles mentioned in the name which I here give it, enter into its composition. It has but little solubility in cold water, but is very soluble in hot, from which it may be obtained in crystals by cooling.

Of protoxide of copper, a white powder insoluble in water and in most acids, decomposable by alkalies or by distillation

with muriate of ammonia, also by mixture with nitrate of per-oxide of iron; when mixed with five times its weight of hyperoxymuriate of potash it explodes, either by heat, friction, the contact of sulphuric acid, or by the electric spark. This explosion is very loud if it take place in a confined space, as in tubes.

Of per-oxide of copper, is best formed by adding sulphuretted chyazate of soda to sulphate of copper, both in a pure state; it is then in the state of a bright pea-green liquid, to which if any deoxidating substance be added, such as sulphurous acid, an alkaline sulphite, or the salts of protoxide of tin or of iron, then the sulphuretted chyazate of protoxide of copper is immediately produced, and precipitates in its usual form as a white powder.

Of protoxide of lead, a soluble salt, the forms of whose crystals, I am inclined to believe, are obtuse rhombs; in a moist atmosphere these crystals slowly deliquesce.

Sulphuretted chyazate of protoxide of iron, a colourless salt, very soluble.

Sulphuretted chyazate of per-oxide of iron, a salt of a beautiful crimson colour, deliquescent, and not obtainable in a solid form without great precautions, and a particular process, which consists in exposing it to an atmosphere artificially dried.

The sulphuretted chyazates of the following metallic oxides appear, by my experiments, to be all very soluble, viz. oxides of tin, bismuth, manganese, zinc, cobalt, nickel, palladium, uranium, molybdenum, and chromium.

The two sulphuretted chyazates that I have analysed are, that of protoxide of copper, and that of barytes.

Analysis of sulphuretted Chyazate of Protoxide of Copper.

A. Twenty grains of sulphuretted chyazate of protoxide of copper were heated to 413° for half an hour, without experiencing the least diminution of weight; consequently this salt does not contain any water.

B. Ten grains of crystallized sulphate of copper were dissolved in water, together with fifteen grains of green sulphate of iron, and were decomposed by sulphuretted chyazate of magnesia; the product was sulphuretted chyazate of protoxide of copper: after washing and drying at 212°, it weighed 4.58 grains. Now as the whole of the copper contained in the ten grains of sulphate entered into the composition of the 4.58 grains of sulphuretted chyazate in the state of protoxide, and as the quantity in those ten grains is according to PROUST 2.56 grains, which reckoned as protoxide is equal to 2.88 grains, it follows that 4.58 grains of the sulphuretted chyazate contain 2.88 grains of protoxide of copper, or 62.85 per cent.

C. Five grains of sulphuretted chyazate of protoxide of copper were decomposed by nitric acid, and the resulting nitrate of copper converted into a muriate, from which the copper was thrown down in the metallic state by iron; it weighed 2.82 grains, which reckoned as protoxide is equal to 3.172 grains, or to 63.44 per cent.

As this salt does not contain any water (A), its only constituents are sulphuretted chyazic acid and protoxide of copper, which exist in it in the following proportions.

	Experiment B.	Experiment C.	Mean of both.
Sulphuretted chyazic acid	37.15	36.56	36.855
Protoxide of copper	62.85	63.44	63.145
	100.00	100.00	100.000

Analysis of sulphuretted Chyazate of Barytes.

A. Ten grains of this salt that had been previously dried at 212° and weighed immediately, were heated to 400° for half an hour without losing any weight.

B. Another ten grains in the same state of dryness, dissolved in water and decomposed by sulphuric acid, gave of ignited sulphate of barytes 10.5 grains, which contain according to BERZELIUS 6.93 of barytes.

C. Another ten grains in the same state, and also dissolved in water, were decomposed by muriate of protoxide of copper; they produced eight grains of sulphuretted chyazate of protoxide of copper, the acid contained in which, by my analysis of that salt, is 2.95 grains.

The sulphuretted chyazate of barytes fully dried at 212° is therefore composed as follows.

	Experiment B.	Experiment C.	Mean of both.
Sulphuretted chyazic acid	30.7	29.5	30.1
Barytes - -	69.3	70.5	69.9
	100.0	100.0	100.0

The soluble sulphuretted chyazates are excellent tests not only for indicating the presence of oxide of copper in a solution, but also for shewing its quantity with great exactness, as

they precipitate the last atom of that oxide from its solutions in the state of an insoluble salt, whose composition is known. In order that it should have this effect however, the oxide of copper should exist in the solution as protoxide, (in which state it is seldom obtained in analyses) or which answers the same purpose, a deoxidating agent, such as sulphurous acid, or its salts, or a salt of protoxide of tin, or of iron, must be added to it.

Other useful applications of these salts and of their acid will also naturally occur to the ingenious analyst who wishes to avail himself of its peculiar actions on metallic bodies. It would be tedious to point these out, and their recital would extend a communication that is already of greater length than I could have wished, and which I here close.

Tower, June 6, 1814.

XXVII. *Some Experiments on the Combustion of the Diamond and other carbonaceous Substances.* By Sir Humphry Davy, LL.D. F.R.S. V.P.R.I.

Read June 23, 1814.

SINCE it has been shewn by various accurate experiments, that the diamond and common carbonaceous substances consume nearly the same quantity of oxygene in combustion, and produce a gas having the same obvious qualities, a number of conjectures have been formed to explain the remarkable differences in the sensible qualities of these bodies, by supposing some minute difference in their chemical composition; these conjectures have been often discussed, it will not be necessary therefore to dwell upon them: M. M. BIOT and ARAGO, from the high refractive power of the diamond, have supposed that it may contain hydrogene; I ventured to suggest in my third Bakerian Lecture, from the circumstance of its non-conducting power, and from the action of potassium upon it, that a minute portion of oxygene may exist in it; and in my Account of some new experiments on the fluoric Compounds,* I hazarded the idea that it might be the carbonaceous principle combined with some new light and subtile element of the class of supporters of combustion. M. GUYTON DE MORVEAU, who conceived he had proved by experiments made fourteen years ago, that common carbonaceous substances were oxides of diamonds, from his last researches, conducted in the same manner as those of Messrs.

* Phil. Trans. for 1814. Part I. p. 72.

ALLEN and PEPYS, seems still inclined to adopt this opinion, though in admitting a much smaller quantity of oxygene than he originally supposed in charcoal; and he considers the diamond as pure carbonaceous matter, containing, possibly, some atoms of water of crystallisation.

I have long had a desire of making some new experiments on the combustion of the diamond and other carbonaceous substances, and this desire was increased by the new facts ascertained with respect to iodine, which by uniting to hydrogene, affords an acid so analogous to muriatic acid, that it was for some time confounded with that substance. My object in these experiments, was to examine minutely whether any peculiar matter was separated from the diamond during its combustion, and to determine whether the gas, formed in this process, was precisely the same in its minute chemical nature, as that formed in the combustion of common charcoal. I have lately been able to accomplish my wishes; I shall now have the honour of communicating my results to the Royal Society.

During a stay that I made at Florence in the end of March and beginning of April, I made several experiments on the combustion of the diamond, and of plumbago, by means of the great lens in the Cabinet of Natural History; the same instrument as that employed in the first trials on the action of the solar heat on the diamond, instituted by COSMO III. Grand Duke of Tuscany; and I have since made a series of researches on the combustion of different kinds of charcoal at Rome, in the laboratory of the Academia Lyncei. In the first series I was honoured by the assistance of the Count BARDI, the Director, and Signior GAZZARI, the Professor of Chemistry at the

Florentine Museum; and in the last by that of Sig. Morrichini and Barlocci, Professors of the College Sapienza at Rome.

In the very first trials on the combustion of the diamond, I ascertained a circumstance that I believe has not been noticed before; namely, that the diamond, when strongly ignited by the lens in a thin capsule of platinum perforated with many orifices, so as to admit a free circulation of air, continues to burn in oxygene gas after being withdrawn from the focus. The light it affords is steady, and of so brilliant a red, as to be visible in the brightest sunshine; and the heat produced is so great, that in one experiment, in which three fragments of diamonds weighing 1.84 grain only were burnt, a fine wire of platinum used for attaching them to the tray was fused, and that some time after the diamonds were removed out of the focus.

The knowledge of this circumstance enabled me to adopt a very simple apparatus and mode of operation in my researches, and to complete, in a few minutes, experiments which have been supposed to require the presence of a bright sunshine for many hours.

My apparatus consisted of clear glass globes of the capacity of from fourteen to forty cubical inches, having single apertures to which stop-cocks were attached; a small hollow cylinder of platinum, which I use in experiments with the blow pipe, was attached to one end of the stop-cock, and was mounted with a little perforated capsule of platinum for containing the diamond. When the experiment was to be made, the globe containing the capsule and the substance to be burned was exhausted by an excellent air pump, and pure oxygene gas, made from hyperoxymuriate of potassa, admitted. The globe

before and after the experiment was brought to the same temperature as the water over which the oxygene gas had remained. And as during the short time required for the combustion there was no sensible change either in the thermometer or barometer, no corrections for pressure or temperature were rendered necessary; the change of volume in the gas after the combustion, was estimated by means of a fine tube connected with a stop-cock, adapted by a proper screw to the stop-cock of the globe, and the absorption was judged of by the quantity of mercury that entered the tube, which afforded a measure so exact that no alteration, however minute, could be overlooked. As the elastic force of the vapour of water is the same at the same temperature, it was evident, that if any water formed in these experiments, it would be deposited as dew or mist in the globe; and I am convinced by direct trials, that a quantity of moisture not capable of being weighed by a balance sensible to the $\frac{1}{100}$ of a grain, is rendered evident by deposition on a polished glass surface.*

The diamonds were always heated to redness before they were introduced into the capsule.

During the combustion of the diamond, the glass globe was kept cool by the application of water to that part of it immediately above the capsule, and where the heat was greatest.

In the first experiment, three diamonds, weighing together 1.63 grain, were entirely consumed, in a quantity of oxygene

* A piece of paper weighing a grain was introduced into a tube of about the capacity of four cubical inches, the exterior of which was gently heated by a candle; immediately a slight dew was perceptible in the interior of the upper part of the tube; the paper taken out and weighed immediately in the balance above referred to, had not suffered any appreciable diminution.

gas, more than three times as much as was necessary to convert them into carbonic acid. In this case, after the combustion had once commenced, it continued without a fresh application of the lens till there remained only a very thin piece of the largest diamond in contact with the capsule, and this by being brought into the focus, rapidly disappeared. On restoring the globe to its original temperature, there was a very evident deposition of moisture; but on arranging the apparatus, so as to ascertain the change of volume of the gas, there entered only twenty-one grains of mercury. In this experiment, the cylinder of platinum had been fastened into the stop-cock by means of a small perforated cork; it seemed probable, when the small diminution of gas was considered, that the appearance of moisture might be owing to the production of vapour from this cork during the combustion, and the second experiment demonstrated that this was the case.

In this second experiment 1.84 grains of small diamonds were employed, and a glass globe of the capacity of 14.9 cubical inches. Soon after the capsule was placed in the focus in bright sunshine, the diamonds burnt with great brilliancy, and continued to burn till they had considerably diminished in bulk; but their splendour of combustion gradually became less, and before they had apparently lost half of their volume the process ceased. By placing them a second time in the focus, after agitating the globe so as to change their places, the combustion was again produced; but the light was much less vivid than before, and the combustion continued for a much shorter time. They were exposed to the concentrated rays a third and a fourth time, but after the fourth time they seemed incapable of burning, and though kept for some

minutes in the focus, appeared to undergo no further diminution: two fragments remained, which, as it was afterwards found, weighed .52 of a grain; the barometer during the experiment was at 29.9 inches, the thermometer at 56° FAHRENHEIT. When the original temperature of the globe was restored, there was not the slightest appearance of vapour or humidity; the interior was as clear as before the experiment, and there was no solid matter of any kind separated in the tray. The fragments of diamond which remained were not black, but had lost their lustre like glass that has been acted on by fluoric acid, nor at any period of the process was any carbonaceous appearance perceived upon them. When the communication was made by the stop-cock between the interior of the globe and a surface of mercury, a minute quantity entered equal to 1.5 grain only.

A portion of the gas in the globe was transferred into a tube in the mercurial apparatus, and the oxygene it contained absorbed by the combustion of phosphorus; 3.5 parts of gas heated in this way left a residuum of 2.5 parts. A portion of the gas was agitated with lime water, when seven parts out of ten were absorbed. I exposed the gas which remained after the combustion of phosphorus to several tests; it had not only the obvious characters of carbonic acid, but exhibited exactly the same chemical phenomena. Potassium strongly heated in it in a small glass tube over mercury, burnt with a dull red light, and formed an alkaline product of the same intense black colour as that produced by its combustion in the carbonic acid procured by the dissolution of marble: distilled water absorbed rather less than its own volume of the gas, and became subacid, sparkled by agitation, gained the taste and smell

of a solution of carbonic acid in water, precipitated in the same manner lime water, and when in excess redissolved the precipitate. To ascertain if this precipitate was exactly the same in composition as pure carbonate of lime, I made a sufficient quantity of it by pouring lime water into the recipient containing the results of the first experiment; and after collecting and drying it at the temperature of 212° FAHRENHEIT, I introduced a quantity of it contained in some foil of platinum through mercury into a glass tube filled with mercury, and I heated in the same manner an equal quantity of finely powdered Carrara marble, and admitted to them equal quantities of solution of muriatic acid. In this trial, there was rather more elastic fluid disengaged from the Carrara marble than from the carbonate of lime from the diamond; but on examining the foil of platinum after the experiment, I found that a little of the carbonate had not been acted upon: I tried two similar experiments, substituting bibulous paper instead of the metallic foils for infolding the carbonates; the results were such as to shew that both substances afforded the same quantities of elastic fluid.

I heated some of the carbonate from the diamond in a tube which contained potassium, and passed the potassium through it in vapour: there was ignition, and a substance of a dense black colour was formed; this substance was acted on by dilute muriatic acid, when it left a fine black powder which burnt like lamp black, and when thrown into fused nitre scintillated and disappeared in the same manner as powdered charcoal.

The gas that remained in the second experiment, after the absorption of the carbonic acid gas, vividly supported combustion, and diminished with nitrous gas; but as the degree of

purity of the oxygene gas with which the globe was filled had not been determined before the experiment, it was impossible to ascertain with precision, that no elastic matter had been emitted during the process. To determine this point, I made a third experiment. A thin diamond weighing .93 of a grain was introduced into the platinum capsule, which was placed in a globe filled with water and inverted in water, some oxygene gas, the last portion from the decomposition of hyperoxymuriate of potassa, was thrown into the globe, so as to displace the water below the level of the capsule. The focus of the great lens was thrown upon the capsule, which with the diamond was instantly rendered dry by it, and the diamond soon entered into combustion and burnt as usual. After the process was finished, the carbonic acid was absorbed by lime water, and the remaining gas, which equalled about one-third of the quantity of oxygene originally used, was compared analytically in several experiments with a portion of the same oxygene as that introduced into the globe, two measures of nitrous gas being added to a measure of each of the gases; the diminution was less by from $\frac{1}{100}$ to $\frac{3}{100}$ parts in the cases in which the gas that had been exposed to the action of the diamond was used; but this minute difference is what might have been expected, and which indeed could not fail to exist, when it is considered that, during the absorption of carbonic acid gas by water and lime water, a small quantity of common air is always expelled from the water.

In this last experiment a small fragment of diamond remained unconsumed, which was similar in appearance to that mentioned in the second experiment, and its colour, which was originally yellow, was rendered rather darker. In no one of

the three experiments was there any distinct appearance of carbonization, when the process was stopped in consequence of the impurity of the gas; though the diamonds were of various colours and different lustres.

A piece of plumbago from Borrowdale in Cumberland, weighing two grains, was exposed in the focus in the same manner as the diamond in the first and second experiments, having been previously heated red; the quantity of oxygene gas employed was 8.5 cubical inches: more than half the plumbago remained unconsumed, and during the combustion some brown ashes were produced. The phenomena in this experiment were very different from those observed in the experiments on the diamond, the gas became clouded during the process, and there was a considerable deposition of dew on the interior of the globe. When the original temperature of the globe was restored, and the stop-cock opened, 96.6 grains of mercury entered, and drops of moisture even were observed condensed on the sides of the vessel.

In the second series of experiments, charcoal formed by the action of sulphuric acid on oil of turpentine, and some produced during the formation of sulphuric ether, from which nitric acid had been distilled, and which afterwards had been strongly ignited, and charcoal of oak which had undergone the same process, were used.

Three grains of the charcoal from turpentine were employed, 2.5 of that from alcohol, and five grains of the charcoal of the oak: in all these instances of combustion the gas became clouded during the combustion, and when the original temperature of the globe was restored, moisture was found condensed in the interior; much the largest quantity in the

experiment on the charcoal of oak, and the least in that on the charcoal procured from oil of turpentine. The charcoal from oak left a residuum of white ashes, which was principally carbonate of lime; that from oil of turpentine produced no residuum; that from alcohol, which was formed in a common process of the manufacture of ether, left a minute quantity of ashes, probably owing to the impurity of the sulphuric acid employed.

The quantity of mercury which entered the apparatus indicating the change of volume of the gas, was in the experiment

on the charcoal of oil of turpentine	-	107.5 grains
In that on the charcoal of alcohol	-	194.5
In that on the charcoal of oak	-	518.3

From the results of these different experiments, it appears evident, that the diamond affords no other substance by its combustion than pure carbonic acid gas; and that the process is merely a solution of diamond in oxygene, without any change in the volume of the gas; for the slight absorption in the second experiment is scarcely more than a compensation for the volume occupied by the diamonds consumed.

It is likewise evident that in the combustion of the different kinds of charcoal, water is produced; and from the diminution of the volume of the gas, there is every reason to believe, that the water is formed by the combustion of hydrogene existing in the charcoal, and experiments which I have referred to, or detailed in my third Bakerian Lecture, prove the presence of hydrogene in common charcoal, and as the charcoal from the oil of turpentine left no residuum, no other cause but the presence of hydrogene can be assigned for the diminution occasioned in the volume of the gas during its combustion.

M. Guyton de Morveau * has noticed the production of water during the combustion of plumbago from Keswick, and from these experiments it is most probable, that it is formed in the process of combustion, for it is unlikely that water should remain in union with plumbago at a red heat; and in the various experiments that I have made on the ignition of plumbago by Voltaic electricity, I have never perceived the separation of any moisture, or the production of any gas; so that it seems most likely that it contains intimately combined hydrogene. It cannot be supposed that water exists in it in union with oxide of iron, for in this case, there would be no obvious cause for the diminution of the volume of the gas; and all analogy is in favour of the conclusion that the iron is in a metallic state.

The general tenor of the results of these experiments is opposed to the opinion, that common carbonaceous substances differ from the diamond by containing oxygene; for in this case they ought to increase and not diminish the volume of oxygene: nor, on the other hand, is it favourable to the supposition that the diamond contains oxygene, for the difference in the quantity of carbonic acid produced in the different experiments, is no more than may be reasonably ascribed to the generation of water, in the combustion of the common carbonaceous substances; and the results of the experiments, to which I have referred in the beginning of this Paper on the action of potassium on the diamond, may be easily accounted for from other circumstances. †

* Annales de Chimie, Tome LXXXIV. page 241.

† See Bakerian Lecture for 1808. Potassium decomposes the silica in glass by being heated in contact with it, and in the case in which equal quantities of potassium

That charcoal is more inflammable than the diamond may be explained from the looseness of its texture, and from the hydrogene it contains; but the diamond appears to burn in oxygene with as much facility as plumbago, so that at least one distinction supposed to exist between the diamond and common carbonaceous substances is done away by these researches.

A fact which I formerly noticed, the blackening of diamond by the long continued action of heated potassium, induced me to suspect in the beginning of these inquiries, that common charcoal might owe its colour, opacity, and conducting power, to the circumstance of its containing minute portions of the metals of the alkalies, or earths, and plumbago to the iron it contains; but when I found that charcoal made from oil of turpentine, which left no residuum in burning, and charcoal precipitated from carburetted hydrogene gas by chlorine, had the same properties, it was necessary to renounce this opinion.

The only chemical difference perceptible between diamond and the purest charcoal, is that the last contains a minute portion of hydrogene; but can a quantity of an element, less in some cases than $\frac{1}{100000}$ part of the weight of the substance, occasion so great a difference in physical and chemical characters? This is possible, yet it is contrary to analogy, and I am more inclined to adopt the opinion of Mr. TENNANT, that the difference depends upon crystallization. Transparent solid bodies are in general non-conductors of electricity, and it is

were long heated in glass tubes, the one in contact with diamonds, the other alone, that in contact with the diamonds must have acted upon a much greater surface of glass.

probable that the same corpuscular arrangements which give to matter the power of transmitting and polarizing light, are likewise connected with its relations to electricity; and water, the hydrates of the alkalies, and a number of other bodies which are conductors of electricity when fluid, become non-conductors in their crystallized form.

The power possessed by certain carbonaceous substances of absorbing gases, and separating colouring matters from fluids, is probably mechanical and dependent upon their porous nature; for it belongs in the highest degree to vegetable and animal charcoal, and it does not exist in plumbago, coke, or anthracolite.

The same conclusions respecting the composition of carbonic acid may be drawn from these experiments, as from those of Messrs. ALLEN and PEPYS, and THEODORE DE SAUSSURE. If the calculations be founded upon the difference of the weights of oxygene and carbonic acid gases, which appears the most exact method, carbonic acid gas will contain, according to the estimate of the mean specific of the gravities of the two gases given by M. THEODORE DE SAUSSURE,* thirty parts of oxygene, or two definite proportions, to 11.3 parts of carbon, or one definite proportion.

Supposing the diminution of the oxygene produced in the experiments on the common carbonaceous substances entirely occasioned by the formation of water, it is easy to calculate the proportions of hydrogene in them; but in the case of plumbago there is probably a diminution of oxygene, from the oxidation of the iron; and it is not certain that the ashes

* Annales de Chimie, Tome LXXI. pag. 261. This estimation is the same as that I have given. Elements of Chem. Phil. pag. 305.

afforded by the charcoal from vegetable substances exist in it in the state of earths and alkalies: and as the quantity of hydrogene varies according to the degree of heat to which charcoal has been exposed, it is almost useless to attempt to assign its proportion in any particular case, especially when the largest portion is so extremely minute.

The nature of the chemical difference between the diamond and other carbonaceous substances may be demonstrated by another process, namely, igniting them in chlorine; when common well burnt charcoal, or plumbago from Cumberland, is intensely ignited in chlorine, white fumes are immediately perceived in consequence of the production of muriatic acid gas by the hydrogene, which acid precipitates the aqueous vapour in the chlorine; but the diamond occasions no such effect. A small diamond, weighing .45 of a grain, was kept in a state of intense ignition by the great lens of the Florentine Museum for more than half an hour, but the gas suffered no change, and the diamond had undergone no diminution of weight, and was not altered in appearance.

Charcoal, after being intensely ignited in chlorine, is not altered in its conducting power, or colour; and this circumstance is in favour of the opinion, that the minute quantity of hydrogene is not the cause of the great difference between the physical properties of the diamond and charcoal.

XXVIII. *Some Account of the fossil Remains of an Animal more nearly allied to Fishes than any of the other Classes of Animals.* By *Sir* Everard Home, *Bart. F. R. S.*

Read June 23, 1814.

The study of comparative anatomy is not confined to the animals that at present inhabit the earth, but extends to the remains of such as existed in the most remote periods of antiquity; among these may be classed the specimen which forms the subject of the present Paper.

That the bones of the elephant, rhinoceros, hippopotamus, crocodile, and of many other animals should be met with in a fossil state in this island, in such numbers as to make it appear that at some distant period they were inhabitants of Great Britain, is perhaps one of the most wonderful circumstances that occurs in the history of the earth.

To discover the changes that have taken place in our globe, which can account for the remains of animals only fitted to live in warm climates being found in so northern a situation; and to explain the circumstance of human bones never having been met with in a fossil state, is the province of the geologist. To examine such fossil bones, and to determine the class to which the animals belonged, comes within the sphere of inquiry of the anatomist, and considerably increases its extent.

This branch of comparative anatomy not only brings to our knowledge races of animals very different from those with which we are acquainted, but supplies intermediate links in the gradation of structure, by means of which the different classes will probably be found so imperceptibly to run into one another, that they will no longer be accounted distinct, but only portions of one series, and show that the whole of the animal creation forms a regular and connected chain.

The fossil remains of animals are too frequently brought under our observation in a very mutilated state; or are so intimately connected with the substances in which they are deposited, that it is difficult to make out the figure of the bones. In the present instance, the pains that have been taken, and the skill which has been exerted in removing the surrounding stone, under the superintendance of Mr. BULLOCK, in whose Museum of Natural History the specimen is preserved, have brought the parts distinctly into view.

This specimen was found upon an estate of HENRY HOST HENLEY, Esq. between Lyme and Charmouth, in Dorsetshire, in a cliff thirty or forty feet above the level of the sea-shore. It had been thrown down by the breaking off of a part of the cliff, and buried in the sand upon the shore, to the depth of nearly two feet. The skull was dug out in 1812, the other parts in the following year, at a distance of some feet.

The cliff is composed of that species of argillaceous limestone called blue lias, in which the fossil bones were deposited. Above the lias there is only a thin layer of black earth.

The figure and appearance of the fossil bones are so accurately shewn in the annexed drawings, (Plates xvii. xviii. xix. xx.) as to make a very particular description of them unnecessary.

The head is four feet long, and all the parts of one side of the skull, and of the upper and under jaw, are very distinct: the vertebræ immediately behind the skull remain in their natural situation respecting the head; many of the other bones have been displaced in a greater or less degree, shewing that the skeleton, before the bones were rendered fossil, had been pressed upon by a considerable weight, which had broken many of them, and entirely destroyed others.

The lower jaw had been forced a little backwards, and the intermediate bone, by which it was attached to the skull, displaced; but a portion of it is seen projecting beyond the base of the lower jaw.

The bony sclerotic coat of the eye on one side is entire; that of the other is forced through the nose, and a part of it is seen in the opening of the nostril, which is enlarged by the bones in which it is situated being broken.

The vertebræ of the back have been twisted, and their spinous processes broken off; one of them in a detached state is preserved sufficiently entire to shew its shape, and the size of the canal which contained the spinal marrow.

The anterior surfaces of the dorsal vertebræ are exposed, and several of the ribs of the opposite side remain in their place, connected to the vertebræ, with their concave surfaces uppermost. Those of the other side are all forced down upon the vertebræ, and squeezed into a mass: the pressure has been so great, as to give many of them a fluted appearance, the middle line being more crushed than the edges; they are not only in close contact with the vertebræ, but are made to follow all the irregularities of their surface, so forcibly have they been beaten in upon them.

One scapula remains nearly in its place, on the outside of the ribs; the other, less perfect in its form, is seen in the mass of blue lias beyond the broken ends of the concave ribs of the opposite side.

In the same situation are several broken fragments of bones cemented together, but not sufficiently entire to have their forms made out; beyond these are three small flattened bones connected to one another, having their natural form.

The number of vertebræ collected, which appear to have formed one connected chain, is sixty, and when these are placed in a line, the skeleton measures about seventeen feet. The dimensions of the largest vertebræ are one inch and a half thick, three inches broad from side to side; and three and a half broad from the anterior surface to the origin of the spinous process.

There are other vertebræ in the same blue lias, but they are much smaller, and there is no evidence of their having belonged to the same animal.

Detached portions of ribs were found so much crushed, that the softer middle part appeared split through; but on a close examination, some of the substance of the rib was found to remain, although a deep sulcus in this way had been formed. This very uncommon appearance is represented in the annexed drawing.

There are other fragments of fossil bones collected from the same place, and evidently deposited at the same time; but as they are too imperfect to enable me to ascertain the exact figure of the bones, and it cannot be determined whether they really belonged to this animal, or to others, I have not ventured to notice them on the present occasion.

The circumstance of the lower jaw extending backwards beyond the skull, and there being an appearance of small conical teeth contained within the large ones, led me to believe that the skull belonged to a species of crocodile; but this conjecture was readily proved to be erroneous by an examination of the intervertebral joint, which is an oval cavity, only met with in fishes. The spinous process of one of the vertebræ which is preserved, shews that the fish was not of the family of sharks or rays.

As no fishes that I am acquainted with, have the young teeth growing up in the cavities of the old ones, it became necessary to ascertain whether these teeth were really of that description. This was done by splitting one of them, and its cavity was found to contain calcareous spar, in appearance only resembling a young tooth; the characteristic mark therefore of crocodile's teeth, which had at first been so very imposing, was thus removed.

Upon examining the mode in which the lower jaw is connected to the skull, there appears to have been no regular joint, as in the crocodile, but a long intermediate flat bone, as in many fishes.

The sclerotic coat of the eye being composed of bone, is a character of the eye of fishes, and not met with in the crocodile. In this animal there is a subdivision into thirteen plates, which is only met with in birds.

The situation of the nostrils, one of which is tolerably distinct, corresponds with those of fishes.

The mode of articulation of the lower jaw with the skull, admits the mouth to be opened to a great extent; in this respect it resembles what is met with in the voracious fishes, of

which I may mention the pike as an example; but in them the teeth are not conical, nor does the lower jaw extend so far back under the skull. The vertebræ and ribs are not unlike those of the pike, but the spinous processes are a great deal shorter.

The jaws and scapulæ, both in shape and size, are more like those of the crocodile than of any fishes at present known; and the three small flat bones near the broken portion of the scapula, have a resemblance to those of the tarsus of a species of turtle.

These particulars, in which the bones of this animal differ from those of fishes, are sufficient to show, that although the mode of its progressive motion has induced me to place it in that class, I by no means consider it as wholly a fish, when compared with other fishes, but rather view it in a similar light to those animals met with in New South Wales, which appear to be so many deviations from ordinary structure, for the purpose of making intermediate connecting links to unite in the closest manner the classes of which the great chain of animated beings is composed.

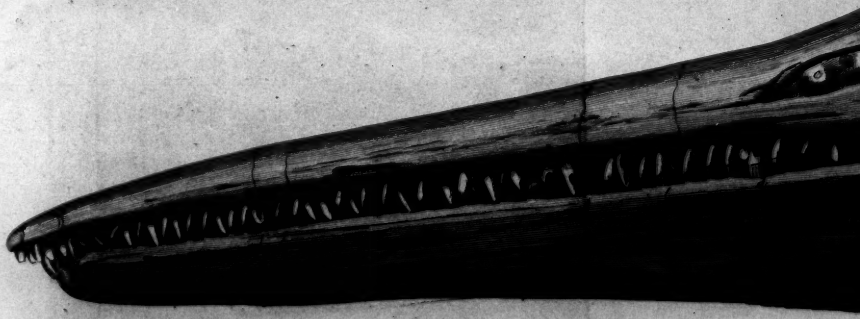
EXPLANATION OF THE PLATE.

PLATE XVII. A side view of the skull and the two jaws.

a a. The blue lias in which the skull was imbedded.

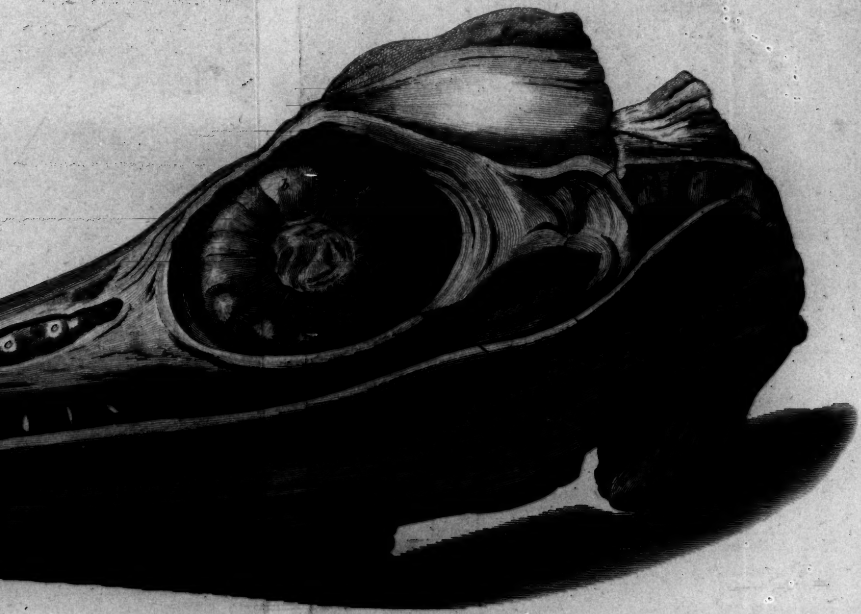
b b. The bony plates in the sclerotic coat of the eye.

c c. The bony plates of the sclerotic coat of the opposite eye forced through the bones of the nose, and only partially exposed.

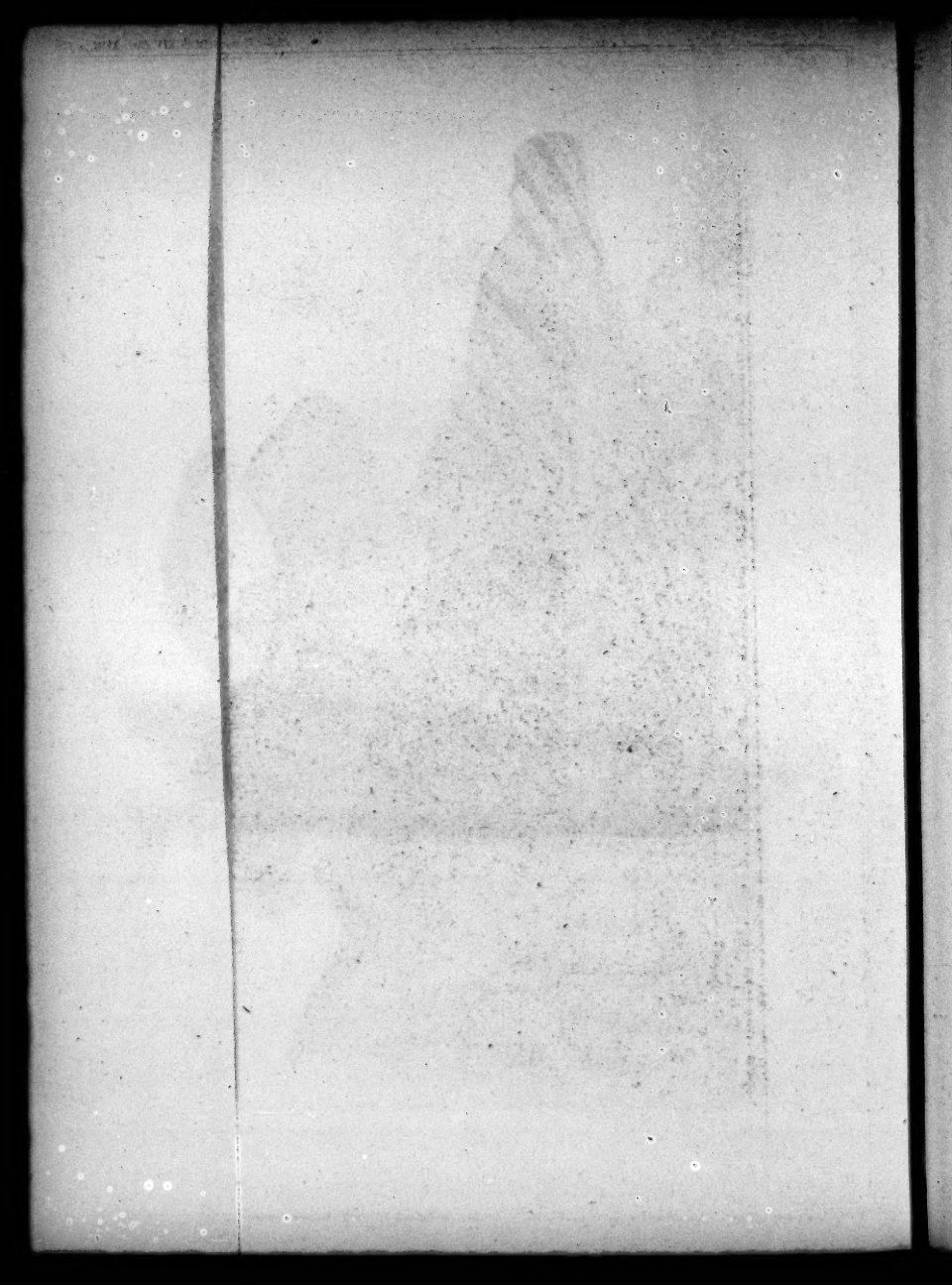


Scale. Three inches

W. Clift del.



inches to a Foot.



Scale. Three Inches to a Foot.

W. Clift del.

F. Chevalier sc.

Natural Size.

W. Clift del.

Size.

Natural Size.

d d. Broken teeth, the cavities filled with calcareous spar.

e e. A portion of the intermediate bone, by which the lower jaw is connected to the skull.

f f f. Three vertebræ connected to the skull.

PLATE XVIII. The continuation of the chain of vertebræ broken off from the mass represented in Plate XVII. The broken surfaces when applied to each other were found to fit exactly.

The anterior surfaces of the vertebræ are exposed, the ribs of the left side are seen upon the convex surface, those opposite on the concave. The left scapula lies over the ribs, a portion of the right scapula is seen detached.

a a. Some of the ribs so much crushed in the middle line of their length, as to appear fluted.

b. The broken portion of scapula.

c, d, e. Bones bearing some slight resemblance to those of the tarsus of the turtle.

f, g. Fragments of bones too imperfect to have their form ascertained.

PLATE XIX. Portions of ribs imbedded in blue lias, to shew the fluted appearance; the parts are represented of the natural size.

PLATE XX. Three vertebræ sawn through in a longitudinal direction to shew the intervertebral cavities which are filled with blue lias. Also one of the spinous processes broken off, in which is seen the canal for the spinal marrow.

a. The detached spinous process.

b b b b b b. Fragments of bones.

c c. The blue lias, in which they are imbedded.

XXIX. *On an easier Mode of procuring Potassium than that which is now adopted.* *By* Smithson Tennant, *Esq. F. R. S.*

Read June 23, 1814.

THE great discovery of Sir H. DAVY, that the alkalies might be decomposed by the Voltaic electricity, was soon succeeded by that of GAY LUSSAC and THENARD, who shewed that a similar decomposition of them might be produced by means of iron.

Besides the new and unexpected fact which was thus brought to light, that the alkaline metals might be deprived of oxygen by a substance inferior to them in attraction; this new process was highly valuable, in affording the means of obtaining them far more abundantly than by electricity.

The circumstances described by GAY LUSSAC and THENARD, as requisite for producing the decomposition of the alkalies by iron, are first, that the iron should be intensely heated, and afterwards that the alkalies should be brought in contact with it in that heated state. For this purpose a furnace must be made, capable of admitting a gun barrel, containing the iron turnings, to pass through it, and a short piece of barrel containing the alkali must be adapted to the former by grinding, so as to be air tight. As this short piece of barrel is out of the furnace, G. LUSSAC and THENARD direct that a separate fire be applied to it, in order to make the alkali pass from it into the longer barrel. To avoid the necessity of a separate fire, this

passage of the alkali has, in England, been generally effected through a small perforation between the two barrels, being poured very hot into the smaller barrel, which is then closed with a ground stopper.

The process conducted in either way requires the construction of a particular furnace, and the correct fitting of the barrels by grinding, so as to be air tight, and being somewhat complicated is not always performed with success.

As it was very desirable to facilitate the mode of obtaining potassium, which is so powerful a chemical agent, I have attempted to simplify the process, and having so far succeeded as to render it capable of being performed in a common smith's fire, and without the junction of the iron barrels by grinding, I have thought it might deserve to be communicated to the Royal Society.

If it was absolutely necessary to heat the alkali and iron separately, and in that state to unite them, no material improvement in the simplicity of the present apparatus could be reasonably looked for; but upon considering that the alkali frequently passed through the short barrel in a few minutes, it did not seem probable that much of the potassium was then formed, since the whole operation required a continuance of the heat for near an hour.

In order therefore to learn whether potassium might not be produced merely by distilling turnings of iron and potash, I put the two together into a piece of gun barrel, the lower end of which was closed by welding, and the upper end by a cork, having a small glass tube through it to admit the escape of air.

The lower end of the barrel being coated as usual with lute

to protect it from the air, was exposed to a strong heat, whilst the upper part was kept cool, and upon opening the barrel, when it had become quite cold, it was found that potassium had sublimed into the upper part. The potassium, however, so produced, though it burnt on contact with water, had not the purely metallic aspect of that formed in the common mode. It was of a more dusky appearance, resembling a mixture of some black powder with potassium. As it seemed probable that some of the potash had risen along with the potassium, I repeated the experiment with attention, to heat the barrel to a greater length, so as to force the potassium to rise further from the ingredients below, but the potassium formed with this precaution had nearly the same dusky appearance as before.

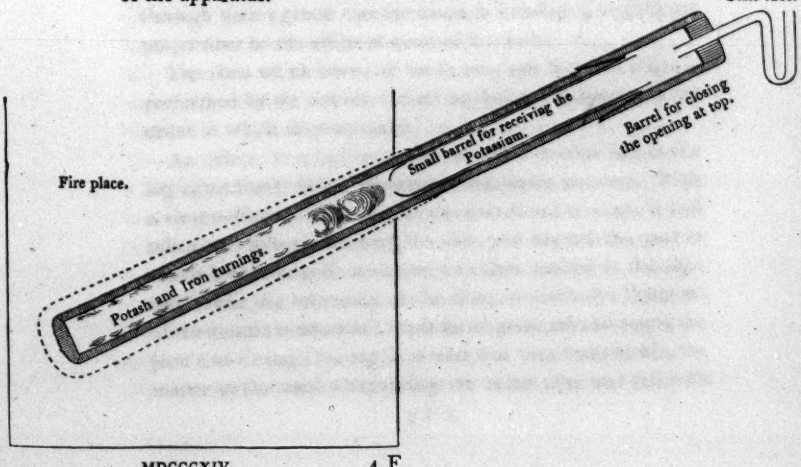
After trying different means for obviating this imperfection, I found the following to be quite effectual. Into the upper part of the barrel a narrower piece, nearly fitting it, was inserted, open only by a perforation at the lower end to admit the vapour of the potassium to pass into it, and upon distilling potash and iron turnings with this addition, the potassium rose into the narrow tube, quite pure, with its usual brightness.

The most convenient dimensions of the apparatus, are for the external barrel to be about a foot and a half long, and the internal one about seven or eight inches. The latter should not be wholly inserted in the former, but about an inch of it left out for the greater ease in withdrawing it. The width is in general determined by that of a common gun barrel, but may be increased to a certain degree. I have had the thick part of a gun barrel so much enlarged by hammering it thinner,

as to contain twice as much iron turnings and potash, and have employed it with success. But on the other hand, there are limits to this extension of the width, arising from the increased difficulty of making the heat penetrate throughout.

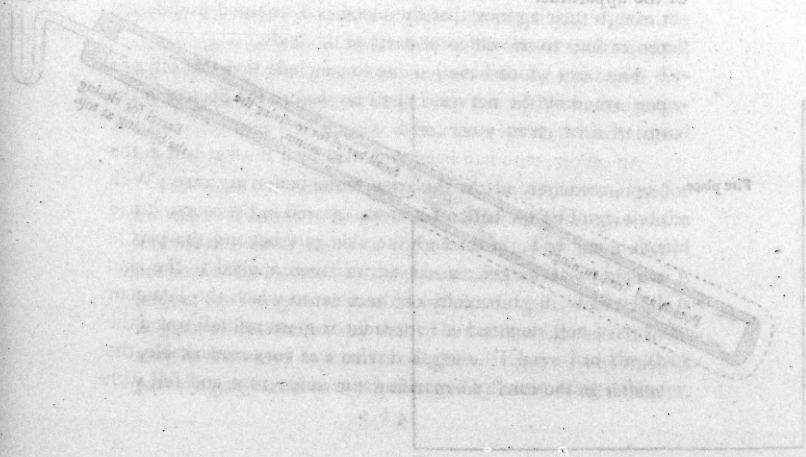
The opening of the barrels at the top must be covered with a cap or wide tube, which being at a distance from the fire need only be fastened with sealing wax, but for the greater security of keeping this part cool, the whole of the tube which is out of the fire should be wrapped round with linen or blotting paper kept wet.

The opening of the wide tube must be closed with a cork having a crooked tube of glass through it, containing a drop of mercury, which being moved by the passage of the air shews that the vessels are perfectly tight. But the annexed sketch will at once shew both the construction and dimensions of the apparatus.


Glass tube.
Small barrel for receiving the Potassium.
Barrel for closing the opening at top.
Fire place.
Potash and Iron turnings.

The principal point to be attended to, both in this and in the common mode, is the giving a strong heat, which should be continued for the greater part of an hour; and to enable the iron barrel to support this, it is quite essential to cover it with a proper lute, carefully applied.

The lute which I have found most effectual for this purpose was composed of a small proportion of Stourbridge clay in its natural state, with a much larger proportion which had before been burnt and powdered, and both of which may be easily had at any glass house.



XXX. *On the influence of the Nerves upon the Action of the Arteries.* By Sir Everard Home, *Bart. F. R. S.*

Read June 30, 1814.

That the pulsations of the arteries correspond in their frequency with the contractions of the left ventricle of the heart, is universally admitted; and those pulsations continuing in the arteries after the limb to which they belong is rendered paralytic, has led to the belief, that all arterial action is independent of nervous influence.

The object of the present Paper is to shew that the nerves which accompany the arteries regulate their actions, and it is through their agency that the blood is distributed in different proportions to the different parts of the body.

The facts which have led me to conclude that this office is performed by the nerves, I shall lay before the Society in the order in which they occurred.

An officer, who had been wounded by a musket ball in the leg immediately below the knee, came under my care. With a view to find the ball, a seton was passed in the course it had taken, and brought through the skin just beyond the part in which it was lodged; a caustic was then applied to the skin just below the tuberosity of the tibia, to which the ligament of the patella is attached: it produced great pain all round the joint and through the leg, and what was very remarkable, the matter in the canal surrounding the seton rose and fell with

great violence; this circumstance induced me to feel the pulse at the wrist, and to keep my finger upon it during the time this effect upon the pus continued, which was several minutes, corresponding with the pulse at the wrist, which, however, had been in no degree disturbed.

The increased action of the arteries, rendered evident by the effect upon the pus, following so closely upon an irritation on the neighbouring nerves, made it clear that it arose from that cause, and brought to my recollection an instance, in which the aorta had been seen pulsating with great violence, in consequence of an irritation upon the nerves of the urinary bladder.

To ascertain whether such a connection between the actions of nerves and arteries could be demonstrated in a state of health, as well as disease, I instituted the following experiments, which were made in the presence of Mr. BRODIE and other gentlemen competent to form a judgment respecting them.

The carotid artery of a dog was laid bare, the intercostal nerve and par vagum, which form one bundle, were separated from it by a flattened probe for one-tenth of an inch in length, the head and neck were then placed in an easy position, and the pulsations attended to by all present, for two minutes, that the eye might be accustomed to them in their natural state: the nerve passing over the probe was then slightly touched with the kali purum. In a minute and a half the pulsations of the exposed artery became more distinct; in two minutes the beats were stronger, and in three more violent; in four minutes the violence was lessened, and in five minutes the action was restored to its natural state.

This experiment was repeated with the same results upon

a rabbit, and in that animal the par vagum was separated from the intercostal nerve, which cannot be readily done in the dog; and it was found when the par vagum alone was irritated, no change took place in the action of the artery. The carotid artery was chosen as the only one in the body of sufficient size that can be readily exposed, to which the nervous branches supplying it can readily be traced from their trunk.

This experiment was repeated three different times, so as to leave no doubt respecting the result.

Having ascertained by these experiments that the increase and diminution of the action of an artery does not depend upon irritability, but nervous influence, I made the following experiments, to determine whether heat or cold had the greatest effect in stimulating the nerves to action.

The wrist of one arm was surrounded by bladders filled with ice, and after having remained in that state five minutes, the pulse was felt at the same time in that and in the opposite wrist, and the beats were found evidently strongest in the cooled wrist. This experiment was then made with water heated to 120° or 130°, beyond which the heat could not be submitted to, and the pulse was found to be softer and weaker than that in the other arm.

When one wrist was cooled and the other heated, the stroke of the pulse in the cooled arm had great force beyond that in the heated one.

These experiments were repeated upon several young men at different ages, and on several women, with an uniform result, and explain the glow produced by the cold bath, and the other beneficial effects of cold bathing, in a more satisfactory manner than has been hitherto done.

This influence of the nerves upon the arteries, throws considerable light upon some of the most important actions in the animal economy. By its means the same arteries, at different times, allow very different proportions of blood to pass through them, and those employed in furnishing blood for the secretions have the supplies regulated, which explains the use of the system of nerves with which the blood vessels of the viscera are so abundantly furnished.

The erection of the penis, produced by a particular state of mind, is one of the effects of this influence of the nerves upon the arteries, and the stoppage of the secretions from the same cause is another of an opposite kind.

The ready supply of blood to a limb by the small anastomosing branches, when the principal arterial trunk is obliterated, depends upon the same cause; and the coagulation of the blood in the arterial trunk leading to a mortified part, arises from the nerves having previously lost their influence over it. On this dominion of the nerves over the actions of arteries depends the growth of the body, the regeneration of parts in those animals in which it occurs, as lizards and others, and the formation of tumours of all the different kinds. The circulation of the blood is therefore no longer to be considered as wholly dependent upon the heart and the elasticity of the arteries, for although by these alone it can be kept up, the action of the nerves is necessary to regulate the distribution of the blood to the different parts of the body, accordingly as supplies are wanted to carry on the necessary operations of the animal economy.

XXXI. *On the Means of producing a double Distillation by the same heat.* By Smithson Tennant, *Esq. F. R. S.*

Read June 30, 1814.

It was first made known by the experiments of Dr. Black, and has since been confirmed by those of Mr. Watt and other philosophers, that the quantity of heat required for raising the temperature of water from fifty degrees to that of the boiling point, is only about a sixth of that which is afterwards required for converting the boiling water into steam. As the steam itself is not hotter than the boiling water, the heat which it had absorbed was said by Dr. Black to be latent; being merely employed in supporting the aerial state which the water had assumed. Whenever this steam is condensed, the heat which was latent again reappears, and in such considerable quantity, that it has been found convenient for various purposes to employ the condensation of steam for heating other bodies.

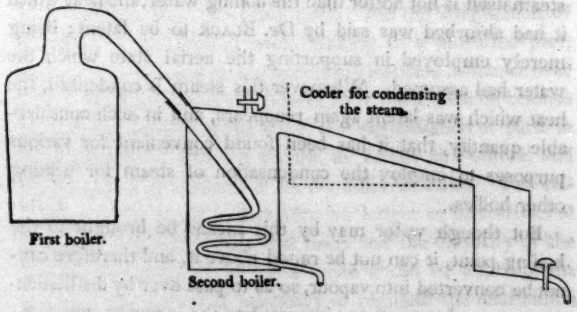
But though water may by this means be brought to the boiling point, it can not be raised above it, and therefore cannot be converted into vapour, so as to pass over by distillation. As soon as the steam has imparted to the water its own temperature, there is no longer any transfer of heat, and the steam passes through the water uncondensed.

If, on the contrary, the steam did continue to condense, then the water would itself be converted into steam, and might by

that means be distilled over without any additional fire; and though this does not take place under the usual circumstances, yet it may be effected in the following manner.

The temperature required for converting any fluid into vapour is dependent on the pressure of the air upon its surface, and may therefore be lowered if that pressure is diminished. If then the weight of the air was removed from water, it would rise into vapour below the common boiling point, and might therefore be distilled over by steam of the usual heat.

In order to produce this effect, a vessel having a receiver connected with it should be made air tight, and the steam made to pass through the vessel along a worm or spiral tube of metal, in the manner represented in the annexed outline.



The vacuum is now easily produced by applying heat to the vessel till the steam issues from the opening in it, and in the receiver, when they are to be immediately closed, and the heat removed.

The water distilled over is collected in the receiver, which is kept cool for that purpose.

An apparatus of this kind I had constructed chiefly for the purpose of explaining the theory of latent heat, or of the capacity of bodies for heat in different states; but it is possible that it may also be of some further practical utility, whenever it is of consequence to economise the consumption of fuel. When water is deficient on board of ship, it has been in some degree supplied by distillation from the ship's boiler, and if the steam from the boiler had been made to pass through the apparatus just described, the quantity would be nearly doubled.

By an experiment which I had made some time ago, about three fourths of the quantity obtained by the first distillation were added by the second. But I believe a larger proportion may be procured when the second distilling vessel is duly coated with flannel, or some light substance, to retain the heat.

Though salt water does not boil at so low a degree of heat as fresh water, yet upon trial with sea water the difference was found to be quite insignificant, compared with that of the steam formed under the usual pressure, and *in vacuo;* and did not sensibly affect the result of the process. The only doubt as to the propriety of taking such a vessel to sea, would arise from the degree of danger which there is of experiencing a want of fresh water. This probably, I apprehend, is not great; but on the other hand, there is the important object of saving the lives of the people in the ship, whenever such deficiency is experienced.

XXXII. *An Account of some Experiments on Animal Heat.* By John Davy, *M. D. F. R. S.*

Read February 17, 1814.

THE recent inquiries of Mr. BRODIE have rendered questionable the different prevailing hypotheses relative to animal heat, and have shewn that fresh investigation is necessary, before we can expect to arrive at any accurate theory.

In the present uncertain state of our knowledge, three circumstances are particularly deserving of attention, viz. the relative capacities of venous and arterial blood for heat, their comparative temperatures, and the temperatures of different parts of the animal body.

On the first of these subjects, we possess only the experiments of Dr. CRAWFORD, which I believe have not yet been repeated, notwithstanding they form the basis of his hypothesis.

On the second, little inquiry has been made, and especially of late years, since the improvement of the thermometer.

And on the third, the observations that have been collected are very few in number, and with the exception of those of Messrs. HUNTER and CARLISLE, are scarcely, perhaps, deserving of confidence.

Such were the inducements that led me to the consideration of each of these subjects apart, and to endeavour to acquire by experiment, some more certain knowledge respecting them.

The experiments that I have made, will be described in the two following sections, and in the last will be offered the few remarks and conclusions which naturally arise, and are fairly deducible from the results.

1. *On the capacities of venous and arterial Blood for Heat.*

I must premise, that my object has been to endeavour to ascertain the relative capacities of venous and arterial blood for heat, rather than their exact specific caloric. The latter, from many circumstances, is difficult to be accomplished; whilst the former is comparatively easy, and in a theoretical point of view is probably equally useful.

I have employed both the methods commonly used. I shall mention most of the experiments that I have made, without noticing the repetitions of them, and shall begin with those on the times of cooling of equal volumes of venous and arterial blood.

The blood used was from the jugular vein and the carotid artery of a lamb, about four months old. It was received in bottles, and to remove the fibrin, which is a great impediment in experiments of this kind, it was immediately stirred with a wooden rod. In respect to colour, the difference between the venous and arterial blood was not so great as in the sheep's, and this in a great variety of instances I have always observed, the venous being of a less dark hue. The specific gravity of the venous blood, without the fibrin, was found to be 1050, and that of the arterial 1047.

A glass bottle equal in capacity to 2518 grains of water, and weighing 1332 grains, was filled respectively with water and venous and arterial blood of the temperature of the room

62, about four hours after the blood had been drawn, during which time each bottle had been closely corked. A delicate thermometer, by means of a perforated cork, was placed in the middle of the liquid. The bottle was then plunged into water of the temperature 140 FAHRENHEIT, and when the mercury had risen to 120, the bottle was quickly wiped and suspended in the middle of the room, and the progress of cooling was noticed every five minutes, till the thermometer had fallen to 80. The following were the general results obtained.

Water cooled from 120 to 80 in	91 minutes
Arterial blood in -	89
Venous blood in - -	88

Considering therefore the capacity of water for heat to be denoted by 1000, neglecting the effect of the glass bottle producing a difference only of about half a minute, and the same in each instance, and dividing the times of cooling by the specific gravity, the relative capacities of venous and arterial blood without fibrin, appear to be as .921 and .934.

In the following experiments the same kind of blood and the same quantity was used as in the preceding. The mixtures were made in a very thin glass receiver containing a elicate thermometer. The temperature of the room was 66.

Hot water temperature 121; cold water 61. Mixture of the two 90, after two minutes 89, after three 88, and after eight 87.

Venous blood 121. Water 62.5. Mixture 89; after three minutes 88.5; after seven 87.

Arterial blood 121. Water 63.5. Mixture 89.5; after three minutes 88.5, and after seven 87.

Now, allowing about one degree of the cooling effect to

have been produced by the receiver indicated by the admixture of the hot and cold water, calculating the quantity of blood used from the knowledge of its volume and specific gravity, employing the formula given by Professor Robison, which consists in multiplying respectively the weight of the water and the blood by the change of temperature, and dividing the first product by the second, the quotient or specific caloric for venous blood appears to be as .812, and for arterial as .814, results very similar to those I have obtained with the blood of the sheep.

In the remaining experiments, blood with the fibrin present was employed, and with this exception they were perfectly similar to those already described.

The blood used to ascertain its time of cooling, was obtained from a sheep; and one day the vein was opened, and on the next the artery. The capacity of the bottle employed exceeded that of the first by one ounce measure of water; but it was equally thin. The air of the room was of temperature 69.

Water cooled from 120 to 80 in 118 minutes
Venous blood in - - 112
Arterial blood in - - 113

And hence, as the latter was of specific gravity 1049, its capacity for heat seems to be as .913, and as the former was of specific gravity 1051, its capacity appears to be .903.

In the following experiment, equal volumes of fluid blood and of water were used; which was easily accomplished by means of a thin bottle with a large mouth, to which a cork was adapted, with a perforation more than sufficiently large to admit the bulb of a very delicate thermometer, and of course to allow, when the bottle was filled to the brim, the excess to

flow out on the introduction of the cork, which was always similarly placed. To retard the process of cooling, the bottle was closely surrounded by a thick layer of what is commonly called cotton-wool. Its capacity was equal to five ounce measures of water, or 2400 grains. It was first filled with cold water, which, when its temperature had been ascertained, was thrown into the receiver before used; it was next filled with hot water of temperature about 110, so that the heat of the glass might be nearly the same as that of the blood; and lastly, when the vein or artery had been opened, the bottle was immediately emptied and filled with blood, the temperature of which was ascertained by the thermometer in less than a quarter of a minute. The mixture now was instantly made, and by the same thermometer the highest temperature after mixture was discovered.

The four following trials were made on the blood of two lambs, both about five months old. The temperature of the air was 60.

Cold water 57.5. Venous blood 100. Mixture 80; after one minute 78.5.

Water 58. Arterial blood 103. Mixture 80; after one minute 79.

Water 58. Venous blood 101.5. Mixture 79; after one minute 78.25.

Water 58. Arterial blood 106.5. Mixture 81; after one minute 80.

The rate of cooling was not noticed after the first minute had elapsed, as the blood then generally began to coagulate. The specific gravity was only ascertained in the two last trials; that of the venous blood was found to be 1050, and that of the

arterial 1049; and hence allowing, as before, one degree of the cooling effect to be produced by the receiver, the capacity of venous blood for heat appears to be .852, and that of arterial blood .889.

It is evident that these trials admit of less accuracy than the preceding; and much more confidence, it appears to me, is due to the third series of experiments, so that if required, I should be inclined to give the numbers thence deduced, as the greatest approximation to the truth.

2. *On the comparative temperatures of venous and arterial Blood, and of different parts of the animal body.*

To endeavour to ascertain the comparative temperature of venous and arterial blood, I have made a considerable number of experiments; some of which on lambs, sheep, and oxen, it will be sufficient for me in this place to describe. In each instance, a long incision was made through the integuments; the jugular vein was laid bare, and the exact seat of the carotid artery found. The vein was then opened, and a small delicate thermometer introduced, and thrust about an inch up the vessel beyond the wounded part, and as the bulb of the instrument was small, the flow of blood was not stopped. When the mercury was stationary, its height was marked. The carotid artery next was divided, and the thermometer was immersed in the current of blood, and left there till it ceased to rise.

The following are the results of five experiments made on lambs, all of which were about three months old. The thermometer in the shade stood at 65.

1.	Venous blood 102.5	Arterial blood 104
2.	—— 104	—— 105
3.	—— 104	—— 105
4.	—— 103.5	—— 105
5.	—— 104	—— 105.

The following results were obtained from three experiments on sheep, whose exact age I could not ascertain. The thermometer in the shade was 60.

1.	Venous blood 103.5	Arterial blood 104.5
2.	—— 102.5	—— 104
3.	—— 103	—— 104.

The experiments on oxen were only two in number. The temperature of the air was 64.

1.	Venous blood 100	Arterial blood 101.5
2.	—— 100	—— 101.

In both instances the oxen were knocked down before the vessels were opened; and as respiration had ceased in consequence of the injury of the brain and spinal cord, no difference of colour, of course, was perceptible between the blood from the jugular vein and that from the carotid artery.

These results, so different from what might have been expected, from the observations of Messrs. COLEMAN and COOPER on the temperature of the two sides of the heart, led me to repeat their experiments. The experiments in which I place most confidence were made on lambs, about four months old, and to these I shall confine myself at present. In each instance the animal was killed by the division of the great vessels of the neck; an opening was made immediately into the thorax, and a very delicate thermometer was introduced into the ventricles of the heart by means of a small incision. The

operation occupied so short a space of time, that in three instances the right auricle had not ceased contracting.

1.	Venous blood	104
	Arterial	105.5
	Rectum	104
	Right ventricle	105.5
	Left	106
2.	Rectum	105
	Right ventricle	105
	Left	106
3.	Rectum	105
	Right ventricle	105.5
	Left	106.5
4.	Rectum	105
	Right ventricle	106
	Left	107.

I cannot well explain the difference which exists between the results of the preceding experiments, and those of Messrs. COLEMAN and COOPER, which are directly opposite. Can the mode in which the animals they experimented on were put to death, be the cause of the want of agreement? In death by asphyxia, there is generally an accumulation of blood in the right ventricle, and in many instances I have observed, when the right ventricle has been distended with blood, little difference of temperature between the two sides of the heart.

To describe all the experiments that I have made to ascertain the temperature of different parts of the animal body, would extend this paper to an unusual length, and there is the less occasion for a long description, as the comparative results were very similar. It will be sufficient therefore here

to notice the observations made on the human body, and on that of a lamb.

That the thermometer might be equally applied to all parts of the surface, its bulb, in form nearly cylindrical, was fixed to a small piece of cork, hollowed and lined with fine wool, and thus half its superficies was applied in each instance. The observations were made on the naked body at seven A. M. immediately after quitting bed. The temperature of the air of the room was 70. The following were the results obtained.

At the central part of the sole of the foot -	90
Between the malleolus internus and the insertion of the tendo Achillis, where the artery is felt -	93
Over the middle of the tibia - - -	91.5
On the middle of the calf - - -	93
Over the popliteal artery at the bend of the knee -	95
Over the femoral artery in the middle of the thigh	94
Over the middle of the rectus muscle - -	91
Over the great vessels in the groin - -	96.5
About a quarter of an inch below the umbilicus	95
Over the sixth rib on the left side where the heart is felt pulsating - - - -	94
Over the same place in the right - -	93
Under the axilla, the whole surface of the bulb being applied - - - -	98

About an hour had now elapsed from the commencement of the experiment. The thermometer again applied to the sole of the foot, rose no higher than 85, five degrees less than at first. A disagreeable sensation of cold was experienced, and particularly in those parts not supplied with large vessels,

and out of the course of the great arteries. The body remained unpleasantly chilly till breakfast had been taken, and then a slight degree of pyrexia was perceived; the heat of surface being increased, the pulse quickened, and the mouth slightly parched. After breakfast, the thermometer was applied to both hypochondriac regions, and the left was found one degree higher than the right.

To ascertain the temperature of different parts of the surface beneath the integuments, the bulb of a thermometer was introduced through small incisions about half an inch between the skin and subjacent parts of a lamb just dead. The heat of the rectum was first ascertained, as a means of marking the rate of cooling, and the different parts were then tried in the following order.

Venous blood in the jugular vein	105.5
Arterial blood from the carotid artery	107
Rectum	105.5
Over the metatarsal bone	97
Over the tarsal bone	90
Over the knee joint	102
About the head of the thigh	103
At the groin	104.

Nearly a quarter of an hour had been occupied in making these observations, and the temperature of the rectum was now found to be 105. The three great cavities were next opened in the order enumerated.

Near the lower part of the liver	106
The substance of the liver	106.5
The substance of the lung	106.5
The left ventricle	107

The right ventricle - - - -	106
The central substance of the brain - -	104
Rectum - - - - -	104.5.

Surprised at the temperature of the brain being lower than that of the rectum, I was led to repeat the experiment. It may be proper to notice a few of the results, as it is a curious circumstance which they confirm. The four experiments I shall mention were made on lambs. As soon as the animal was dead, the cranium was perforated and a delicate thermometer introduced into the central part of the brain.

1. Brain 104 Rectum 104.75
2. Brain 104.75 Rectum 105.5
3. Brain 105.5 Rectum 106.5
4. Posterior part of the brain 105.5; anterior 103. Rectum 106.5.

The temperature of the air at the time was 68. Different parts of the brain were found to vary considerably in temperature; the anterior, as already noticed, being lower than the posterior, and the superficial than the deep seated parts.

3. *Remarks and Conclusions.*

That there is no material difference between venous and arterial blood in respect to specific caloric, excepting what arises from difference of specific gravity; that the temperature of arterial blood is higher than that of venous; and the temperature of the left side of the heart, than that of the right; and lastly, that the temperature of parts diminishes as the distance of the parts from the heart increases—are the general results of the preceding experiments.

Admitting the accuracy of these experiments, and I think

that they will be found correct when repeated, what are their consequences in a theoretical point of view?

They are evidently in direct opposition to Dr. CRAWFORD's hypothesis; the essence of which is, that the capacity of arterial blood for heat is greater than that of venous, that there is no difference of temperature between the two ventricles of the heart, and in fact that the heat of all parts is nearly the same.

They are more agreeable to, and indeed they even support the hypothesis of Dr. BLACK, that animal heat is produced in the lungs, and distributed over the whole system by means of the arterial blood.

Neither are they inconsistent with that hypothesis which considers the production of animal heat as dependent on the energy of the nervous system, and arising from all the vital actions constantly occurring.

Besides the results of the preceding experiments, many arguments may be advanced in opposition to Dr. CRAWFORD's hypothesis.

As we never perceive a difference of capacity in bodies without a difference of form or composition; and as very slight differences of the former result only from great changes of the latter, it might be expected *a priori*, as no difference, excepting that of colour, has been detected between venous and arterial blood, that their specific caloric would be very similar. From analogy also, it might have been expected, that the capacity of arterial blood for heat would be much less than that of water, as water appears to exceed almost every other fluid, and as the capacity appears to diminish as the inflammability of compounds increases. But the strongest arguments

against this hypothesis are to be derived from the recent experiments of Mr. BRODIE, and those of M. M. DELAROCHE and BERARD.

Dr. BLACK's hypothesis appears to me far more satisfactory than Dr. CRAWFORD's, and capable of explaining a much greater number of phenomena; but there are objections even to this hypothesis which must be removed, before it can with propriety be received.

The last hypothesis, which I mentioned, that which refers animal heat to vital action, has many facts in its support, and especially the results of Mr. BRODIE's curious and interesting experiments; and the results of my inquiry, as I have already observed, are not incompatible with it. It may be said, that the viscera of the thorax and abdomen are of highest temperature, because these parts are, as it were, the elaboratories of life; and that the heat of the arterial blood, and of the parts best supplied with this fluid, is greatest, because they lie deepest and abound most in the principle of life or vital action. This explanation was suggested to me by my brother Sir H. DAVY. There are some facts which I have observed agreeable to it, but not more so than to the hypothesis of Dr. BLACK. I have found the stomach of the ox, the pyloric compartment, of a higher temperature than the left ventricle itself; thus when the latter immediately after death was 103, the former full of food was 104.5. I have also found the temperature of young animals, in whom all the vital actions are most energetic, higher than that of animals arrived at maturity. I may mention here, in illustration of this statement, a few observations made on infants, as I am not acquainted with any yet published. In one instance I found the heat under the axilla of

a child just born 98.5; after twelve hours 99, and after three days, the same; during the whole of which time it appeared in perfect health. On five other children of the same age, similar observations were made. In two instances of weak infants, the temperature, one hour after birth, was found not to exceed 96, which is two degrees below the standard heat of man in a state of health; but their respiration was still languid, and the next day the heat of the axilla had risen in one to 98.5, and in the other to 99.*

To conclude, as in each hypothesis examined, difficulties are found to exist from facts or the results of experiments of an unbending nature, we must at present either suspend theory altogether and search for *experimenta crucis*, or adopt that hypothesis which is conformable to the greater number of facts. The first measure is certainly most philosophical; but to the latter we are naturally most inclined, and if I were questioned which view is preferable, I should make no hesitation in selecting Dr. BLACK's, which to me appears both most simple and most satisfactory.

* The opinion of HALLER, I am well aware, is contrary to these results, as is expressed in the following paragraph: "Viri feminis calidiores duriori nempe sunt "fabrica, contra pueri aliquanto minus calent quam adulti homines, ut modo natus "puer vix calorem conservet, nisi sollicite et copiose vestibus textis."—Elem. Phys. ii. p. 297. As this great physiologist seems to have drawn his inference merely from the circumstances mentioned, it can have little force, except from the authority of the author; to which may be opposed an equal authority, not less than that of HIPPOCRATES himself: he says, in his fourteenth aphorism, "Quæ crescunt plurimum "habent calidi innati: senibus autem paucus calor."

ADDITIONS AND CORRECTIONS.

Page 135, last line but two, *for* $\sigma \frac{dx}{d\tau}$, *read* $\sigma \frac{dx^\circ}{d\tau}$

——— one, *for* $\sigma \frac{dy}{d\tau}$, *read* $\sigma \frac{dy^\circ}{d\tau}$

———, *for* $\sigma \frac{dz}{d\tau}$, *read* $\sigma \frac{dz^\circ}{d\tau}$

—— 141, line 1, *for* $\sigma' \rho'^\circ \cos. \lambda^\circ \sin. (\epsilon^\circ - \kappa)$, *read* $\sigma' \rho'^\circ \cos. \lambda^\circ \sin. (\epsilon^\circ - \kappa)$

—— 151, —— 1, *for* $\frac{(r^2 - r'^2)^2}{(r+r'^2)}$, *read* $\frac{(r^2 - r'^2)^2}{(r+r')^2}$

—— 180, —— 9, *for* $\sigma' \rho'$ cos. λ° sin. $(\epsilon^\circ - \kappa)$, *read* $\sigma' \rho'^\circ$ cos. λ° sin. $(\epsilon^\circ - \kappa)$

—— 395, line 11, *for* perfectly, *read* perfectly flat

———— 18, *for* S, *read* S'

———— 22, *for* s, *read* s', and *for* rs, *read* rs'

—— 402, table, col. 3, *for* 4° 37', *read* 3° 37'

—— 403, line 12, *for* Ms'Ni, *read* Ms'Nt'

—— 487, —— 3, at the word iodine, add the following note: *I am obliged to M. Courtois for the iodine employed in these experiments, who, with great liberality, furnished me with a considerable quantity.*

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